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JULY 1967

EXACT SIMILAR SOLUTIONS OF THE LAMINAR BOUNDARY-LAYER EQUATIONS

C. Forbes Dewey, Jr., and Joseph F. Gross

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167

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PREFACE

The purpose of this Memorandum is to present a comprehensive and accurate table of values for the similar solutions of the laminar boundary-layer equations for a homogeneous gas. These values may be used to determine skin friction, heat transfer, and appropriate boundary-layer thickness parameters for a wide variety of physical situations. They are applicable to such problems as stagnation-point flows, flow over a wedge or a flat plate, and hypersonic viscous interaction, and may also be used as good approximation for the determination of nonsimilar flows. Well over 1800 solutions are tabulated, of which about five-sixths have not been published before. The study is part of RAND's work for ARPA on reentry aerodynamics. C. Forbes Dewey, Jr. is an Assistant Professor in the Department of Aerospace Engineering Sciences at the University of Colorado and a consultant to The RAND Corporation.

SUMMARY

The purpose of this Memorandum is to present a comprehensive and accurate table of values for the similar solutions of the laminar boundary-layer equations for a homogeneous gas. These values may be used to determine skin-friction, heat-transfer, and appropriate boundary-layer-thickness parameters for a wide variety of physical situations. They are applicable to such problems as stagnation-point flows, flow over a wedge or a flat plate, and hypersonic viscous interaction, and they may also be used as good approximations for the determination of nonsimilar flows.

The general boundary-layer equations for a thermally and calorically perfect binary gas mixture are derived together with a discussion of the numerical integration procedure used to obtain solutions to the equations, the concept of local similarity, and solutions for large values of the pressure-gradient parameter.

The tabulated solutions include effects of pressure gradient ($0 \le \beta \le 5$ and $\beta = \infty$), Mach number ($0 \le U_\infty^2/2H_e \le 1$) temperature-viscosity law ($0.5 \le \omega \le 1$ or $0 \le s \le 0.2$), leading-edge sweep ($0.1 \le t_s \le 1$) wall temperature ($0 \le t_w \le 2$), Prandtl number ($0.5 \le Pr \le 1$), local streamwise velocity ($0 \le u_e^2/u_\infty^2 \le 1$), and mass transfer at the surface ($-1.6 \le f_w \le 0$).

ACKNOWLEDGMENT

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We have been privileged to receive a review of this manuscript by Nelson H. Kemp of the AVCO/Everett Research Laboratory, Everett, Massachusetts.

CONTENTS

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SYMBOLS

$A_1 \ldots A_6$ = constants defined in Eqs. (105), (115), and (120)

a = function defined in Eq. (109); also function defined in Eq. (154)

B = function defined in Eq. (86)

C = Chapman-Rubesin constant

$C(a)$ = limit of $\tilde{g}(a,t)$ as $t \to \infty$

C_f = local skin-friction coefficient; $C_f = 2\tau/\rho_\infty U_\infty^2$

C_h = local heat transfer coefficient; $q_w/\rho_\infty U_\infty(H_{aw} - H_w)$

c_i = mass concentration of i^{th} component

c_{p_i} = specific heat of the i^{th} component

D_{12} = binary diffusion coefficient

E = function as defined in Eq. (102)

$\mathcal{E}$ = numerical difference function defined in Eq. (82)

e = function as defined in Eq. (86)

F_1 = similarity function defined in Eq. (23)

f' = transformed fluid velocity as defined in Eq. (12)

f_w = wall constant defined by Eq. (22)

$\mathfrak{F}$ = function defined in Eq. (83)

G = dimensionless enthalpy function, (H/H_e)

g = transverse-velocity function, w/w_e

$\mathfrak{G}$ = numerical difference function defined in Eq. (84)

H = total enthalpy of mixture = $h + (1/2)(u^2 + w^2)$

$\mathcal{H}$ = numerical difference function defined in Eq. (85)

h_i = chemical enthalpy of i^{th} component

I_1, I_2, $I_1(1)$, $I_1(2)$, $I_1(3)$ = integrals defined by Eqs. (74) to (78)

$J_n(t)$ = function defined as the n^{th} integral of the error function

j = geometrical index in boundary layer equations; also Reynolds analogy ratio $2C_n/C_f$

k = thermal conductivity of the mixture

Le = Lewis number $\rho D_{12} c_p/k$

M = Mach number

m = constant exponent defined in Eq. (34)

Pr = Prandtl number, $c_p \mu/k$

p = fluid pressure

q = heat flow

R = gas constant

r = radial coordinate defined in Fig. 1

S = Sutherland constant

Sc = Schmidt number, $\mu/\rho D_{12}$

s = S/T_o

s_o = transformed velocity function defined in Eq. (150)

T = temperature of the fluid

T_o = free-stream stagnation temperature

t = transformed similarity variable defined in Eq. (150); transformed variable defined by Eq. (108)

t_{aw} = dimensionless adiabatic wall temperature, $t = t_{\varepsilon w}$ for $\theta'(0) = 0$

t_r = Eckert reference temperature defined in Eq. (53c)

t_s = dimensionless sweep parameter defined by Eq. (32) $= 1 - (U_\infty^2/2H_e) \sin^2 \Lambda$

t_w = dimensionless wall enthalpy ratio defined by Eq. (31) = T_w/T_o

U_∞ = free-stream velocity

u = flow velocity in the x-direction

v = flow velocity in the y-direction

W = constant defined in Eq. (141)

w = flow velocity in the z-direction (transverse velocity)

x = coordinate in direction of flow

y = coordinate normal to the surface

Z_i = dimensionless concentration function defined in Eq. (15)

z = dimensionless function, $(1 - Z_1)$; also, coordinate transverse to the flow direction

β = pressure gradient parameter defined in Eq. (34b)

γ = adiabatic constant = c_p/c_v

δ^* = boundary layer displacement thickness defined by Eq. (80)

ϵ = asymptotic parameter defined in Eq. (98)

η = similarity variable defined in Eq. (11)

Θ = boundary-layer momentum thickness defined in Eq. (81)

θ = dimensionless enthalpy function, $(H - H_w)/(H_e - H_w)$

κ = trial function for solution of first-order equation defined by Eq. (122)

Λ = sweep angle

λ = dimensionless density-viscosity product, $(1/C)(\rho\mu/\rho_e\mu_e)$

μ = viscosity

ξ = transformed x-coordinate defined in Eq. (10)

ρ = fluid density

σ = hypersonic parameter $(U_\infty^2/2H_e)$

σ_1 = modified hypersonic parameter, $(U_\infty^2/2H_e) \cdot (u_e/u_\infty)^2$

σ_2 = modified hypersonic parameter,
$= (U_\infty^2/2H_e)[(u_e/u_\infty)^2 \cos^2 \Lambda + \sin^2\Lambda]$

τ = total skin friction

τ_z = skin friction in spanwise direction

τ_x = skin friction in x-direction

Φ = function defined in Eq. (105)

ζ = variable defined by Eq. (141)

$\chi(a)$ = function defined in Eq. (112)

χ_o = constant defined in Eq. (141)

$\psi(x,y)$ = stream function defined by Eq. (9)

$\psi_1 \ldots \psi_3$ = functions defined in Eqs. (117), (118), and (121)

ω = exponent in the viscosity-temperature law, $\mu \sim T^\omega$

ω_r = constant defined in Eq. (53b)

Subscripts

$(\ )_e$ = function at the edge of the boundary layer

$(\ )_w$ = function at the wall

$(\ )_\infty$ = function evaluated in the free stream

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I. INTRODUCTION

During the last decade, many similar solutions to the laminar boundary-layer equations have been obtained. Most of these results have been for specific exact conditions, such as stagnation-point flow or flat-plate flow, and are based on the assumptions of a linear temperature-viscosity law and a Prandtl number, Pr, of unity. It is well recognized, however, (cf. the citations of Refs. 1 to 5) that an accurate estimate of skin friction, heat transfer, and boundary-layer thickness in compressible flow usually requires the use of a realistic temperature-viscosity law and retention of the dissipation term which appears in the energy equation.

The purpose of this Memorandum is to present comprehensive, systematic, and accurate tables of solutions to the laminar boundary-layer similarity equations for a perfect homogeneous gas. These solutions are applicable to the classical cases wherein the requirements for similarity are satisfied exactly. The solutions may be used also in applying local similarity methods to cases that do not meet the exact requirements for similar flows. In particular, the sensitivity of the numerical values of the wall derivatives (which govern heat transfer and skin friction) and the values of the integral thickness parameters to changes in the similarity variables may be accurately determined. For many physical situations, relaxing one or more of the conditions required for exact similarity will not lead to large errors when the local similar solutions are used to estimate boundary-layer properties.

Solutions are included to illustrate the effects of leading-edge sweep, mass transfer at the surface, pressure gradient, wall temperature,

free-stream Mach number, local external flow velocity, the Prandtl number, and the viscosity-temperature law of the fluid. The tables include most of the numerical solutions obtained by previous authors (recomputed to the accuracy of the present program) and also new solutions. These tables do not include solutions with $\beta < 0$ (decelerating flows) or $f_w > 0$ (suction).

Section II presents the similarity equations in their general form, including the equation for species concentration. Transformations applicable to compressible flows are given, as are equations for the computation of skin friction, heat transfer, and integral boundary-layer thicknesses. Classes of reduced equations are also discussed; these classes represent cases where one or more of the parameters Pr, ω, β, t_s, t_w, $U_\infty^2/2H_e$ and u_e^2/u_∞^2 are either zero or unity.

Section III gives a concise guide to the solutions of this Memorandum. Section IV presents the numerical method of computation used in this program. The concept of local similarity is discussed in Section V, where criteria are presented for judging the applicability of similar solutions in situations where the similarity parameters vary. The analysis is extended to large positive values of the pressure-gradient parameter, β, in Section VI.

Our experience indicates that values of the skin-friction and heat-transfer derivatives and the boundary-layer thickness integrals may be estimated to high accuracy for intermediate values of the similarity parameters by careful cross-plotting of the present exact solutions. The accuracy varies with the particular quantity and the specific values of the similarity parameters, but it is generally between 0.5 percent and 2.0 percent. In particular, use of the solutions for $\beta \to \infty$ allows estimation of all quantities for all positive β.

II. SIMILARITY EQUATIONS FOR THE LAMINAR BOUNDARY LAYER

The purposes of this section are three. First, we shall display the laminar-boundary-layer equations for a binary mixture of perfect gases in their most general form, listing the general conditions that lead to similar solutions. Second, the specific equations treated here are deduced from the general equations, and relations are given for computing heat transfer, skin friction, and integral-thickness parameters. Finally, special reduced forms of the general equations (Pr = 1, etc.) are discussed.

GENERAL EQUATIONS

The equations that describe the conservation of mass, momentum, and energy in a laminar boundary layer are well known. They can be expressed as follows:

$$\frac{\partial}{\partial x}(\rho u r^j) + \frac{\partial}{\partial y}(\rho v r^j) = 0 \tag{1}$$

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \frac{1}{r^j}\frac{\partial}{\partial y}\left(\mu r^j \frac{\partial u}{\partial y}\right) \tag{2}$$

$$\rho u \frac{\partial w}{\partial x} + \rho v \frac{\partial w}{\partial y} = \frac{1}{r^j}\frac{\partial}{\partial y}\left(\mu r^j \frac{\partial w}{\partial y}\right) \tag{3}$$

$$\frac{\partial p}{\partial y} = 0 \tag{4}$$

$$\rho u \frac{\partial H}{\partial x} + \rho v \frac{\partial H}{\partial y} = \frac{1}{r^j} \frac{\partial}{\partial y}\left(r^j \frac{\mu}{Pr} \frac{\partial H}{\partial y}\right) + \frac{1}{r^j} \frac{\partial}{\partial y}\left[r^j \mu \left(1 - \frac{1}{Pr}\right) \frac{\partial}{\partial y}\left(\frac{u^2 + w^2}{2}\right)\right]$$

$$+ \frac{1}{r^j} \frac{\partial}{\partial y}\left[r^j \mu \frac{1}{Pr}\left(1 - \frac{1}{Le}\right) \sum_i h_i \frac{\partial c_i}{\partial y}\right] \tag{5}$$

$$\rho u \frac{\partial c_i}{\partial x} + \rho v \frac{\partial c_i}{\partial y} = \frac{1}{r^j} \frac{\partial}{\partial y}\left(r^j \rho D_{12} \frac{\partial c_i}{\partial y}\right) \tag{6}$$

$j = 0$ two-dimensional flow (7a)

$j = 1$ axisymmetric flow (7b)

The coordinates x, y, and r are defined in Fig. 1. Other symbols are defined as follows:

c_i = mass fraction of i^{th} constituent

D_{12} = binary-diffusion coefficient

H = total enthalpy of mixture = $h + (1/2)(u^2 + w^2)$

h_i = chemical enthalpy of i^{th} constituent

k = thermal conductivity of fluid

Le = Lewis number, $\rho D_{12} cp/k$

Pr = Prandtl number, $c_p \mu/k$

p = local fluid pressure

Sc = Schmidt number, $\mu/\rho D_{12}$

u,v = flow velocities in the x, y directions, respectively

w = flow velocity in direction normal to x, y plane

μ = viscosity coefficient of fluid

ρ = local fluid density

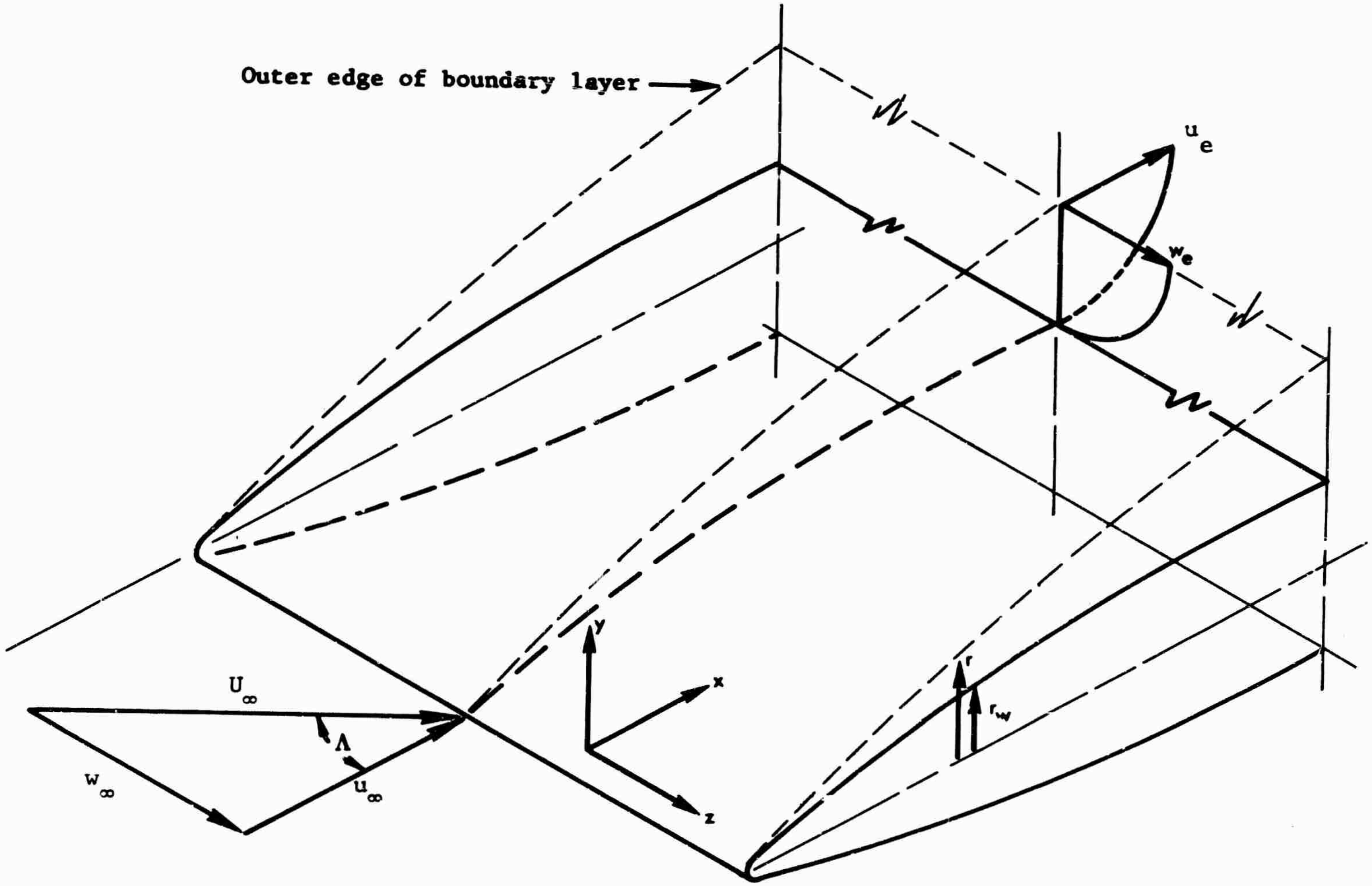

Fig. 1 -- Boundary-layer coordinate system.

These equations neglect thermal diffusion and diffusion-thermo effects.

If the fluid is a mixture of thermally perfect gases, Eq. (4) integrates to

$$p(x, y) = p_e(x) = \rho_e R T_e \tag{8}$$

A stream function $\Psi(x,y)$ is introduced that satisfies the continuity equation:

$$\partial\Psi/\partial x = - \rho v r^j \tag{9a}$$

$$\partial\Psi/\partial y = + \rho u r^j \tag{9b}$$

and a transformation for the independent variables is introduced:

$$\xi = \int_0^x C \rho_e \mu_e u_e r_k^{2j} \, dx \tag{10}$$

$$\eta = \frac{u_e}{\sqrt{2\xi}} \int_0^y \rho r^j \, dy \tag{11}$$

where subscript e denotes any property at the edge of the boundary layer and r_k denotes a characteristic radius. The quantity C, by definition, is a combination of physical properties that are functions of x only.

In reducing the partial differential equations in η and ξ to ordinary differential equations in the single variable η, all derivatives with respect to ξ within the boundary layer are neglected. The fluid velocity, the enthalpy, and the concentration are then defined in terms of the transformed similarity variable η:

$$u/u_e = df(\eta)/d\eta = f'(\eta) \qquad (12)$$

$$w/w_e = g(\eta) \qquad (13)$$

$$H/H_e = G(\eta) \qquad (14)$$

$$c_i/c_{iw} = Z_i(\eta) \qquad (15)$$

(The dependent variable f appearing in Eq. (12) is simply equal to $\sqrt{2\xi}$ times the stream function ψ.) Then the boundary-layer equations, (2), (3), (5), and (6) for a two-component boundary layer may be written as a set of coupled ordinary nonlinear differential equations.

$$\left(\lambda \frac{r^{2j}}{r_k^{2j}} f''\right)' + ff'' = \frac{2\xi}{u_e}\frac{du_e}{d\xi}\left(f'^2 - \frac{\rho_e}{\rho}\right) \qquad (16)$$

$$\left(\lambda \frac{r^{2j}}{r_k^{2j}} g'\right)' + fg' = 0 \qquad (17)$$

$$\left(\lambda \frac{r^{2j}}{r_k^{2j}} \frac{G'}{Pr}\right) + fG' = \left\{\left[\lambda \frac{r^{2j}}{r_k^{2j}}\left(\frac{1}{Pr} - 1\right)\right] \frac{U_\infty^2}{2H_e} 2\left[f''f'\left(\frac{u_e}{u_\infty}\right)^2 \cos^2\Lambda + gg'\sin^2\Lambda\right]\right\}'$$

$$+ \frac{c_{1w}}{H_e}\left[\lambda \frac{r^{2j}}{r_k^{2j}} \frac{1}{Sc}\left(\frac{1}{Le} - 1\right)(h_1 - h_2)Z_1'\right]' \qquad (18)$$

$$\left(\lambda \frac{r^{2j}}{r_k^{2j}} \frac{1}{Sc} Z_1'\right)' + fZ_1' = 0 \qquad (19)$$

where

Λ = leading-edge sweep angle

λ = $(1/C)(\rho\mu/\rho_e\mu_e)$

u_∞ = free-stream flow velocity in the x-direction

The above equations are applicable to the two-component boundary layer of an axisymmetric body or a swept two-dimensional surface. The boundary conditions for these equations are given at the surface of the body by the no-slip condition and the requirement of similarity for the wall concentration and enthalpy:

$$f'(0) = 0 \qquad (20a)$$

$$g(0) = 0 \qquad (20b)$$

$$G(0) = G_w \qquad (20c)$$

$$Z_1(0) = 1 \qquad (20d)$$

At the outer edge of the boundary layer, the velocity and enthalpy must match the values in the free stream. The concentration of any material injected at the surface is assumed to be zero in the external flow.

$$f'(\infty) = 1 \qquad (21a)$$

$$g(\infty) = 1 \qquad (21b)$$

$$G(\infty) = 1 \qquad (21c)$$

$$Z_1(\infty) = 0 \qquad (21d)$$

The final boundary condition is obtained by considering the 2-particles at the wall. This yields the well known Eckert-Schneider condition represented by the equation:

$$f_w = f(0) = \left[\frac{\lambda}{Sc}\right]_w \frac{c_{1w}}{1 - c_{1w}} z_1'(0) \left(\frac{r_w}{r_k}\right)^2 \qquad (22a)$$

From the continuity equation, the injection parameter is related to the similarity coordinates by the equation

$$\left.\begin{aligned} f_w &= - \frac{(\rho v)_w \sqrt{2\xi}}{C\rho_e \mu_e u_e} \cdot \frac{r^j}{r_k^{2j}} \quad \text{all } \beta \\ f_w &= - \frac{v_w}{u_e} \cdot \frac{r^j}{r^{2j}} \left[\frac{t_s}{\beta} \left(\frac{T_o}{T_e}\right) \frac{1}{u_e} \frac{du_e}{dx} \frac{\mu_w^2}{C\rho_e \mu_e u_e r_k^{2j}}\right]^{-1/2} \quad \beta \neq 0 \end{aligned}\right\} \qquad (22b)$$

CONDITIONS FOR SIMILARITY

In order that Eqs. (16) through (19) be similar, severe restrictions must be placed on some of the terms. These are:

$$\frac{2\xi}{u_e} \frac{du_e}{d\xi} \left(f'^2 - \frac{\rho_e}{\rho}\right) = F_1(\eta) \qquad (23)$$

$$\lambda = \lambda(\eta) \qquad (24)$$

$$r^2/r_k^2 = F_2(\eta) \qquad (25)$$

$$\left.\begin{aligned} &\text{either } Pr = 1 \\ &\text{or } u_e^2/u_\infty^2 = \text{const. and } Pr = Pr(\eta) \end{aligned}\right\} \qquad (26)$$

$$Le = Le(\eta) \tag{27}$$

$$G_w,\ c_{1w},\ \text{and}\ r_w^2/r_k^2\ \text{are constants} \tag{28}$$

$$f_w = f(0) = \text{const.} \tag{29}$$

Equations (23) and (25) represent the most severe restrictions on the system because the external velocity distribution and the body surface must satisfy the requirements shown. It may be verified that Eq. (23) has the following form:

$$F_1(\eta) = \frac{2\xi}{u_e}\frac{du_e}{d\xi}\frac{T_o}{T_e}t_s\left\{f'^2 - \frac{1}{t_s}\left[(1 - t_w)\theta - (1 - t_s)g^2 + t_w\right]\right\} \tag{30}$$

where

$\theta = (H - H_w)/(H_e - H_w)$

T_o = free-stream stagnation temperature

and

$$t_w = H_w/H_e = T_w/T_o \tag{31}$$

(adiabatic and calorically perfect inviscid flow) and

$$t_s = \frac{1 + \frac{\gamma - 1}{2}M_\infty^2\cos^2\Lambda}{1 + \frac{\gamma - 1}{2}M_\infty^2} = 1 - \left(\frac{U_\infty^2}{2H_e}\right)\sin^2\Lambda \tag{32}$$

It should be noted that either $j = 0$ and $0 \le t_s \le 1$, or $j = 1$ and $t_s = 1$. If t_s and t_w are considered to be free parameters, Eq. (30) requires that

$$\frac{2\xi}{u_e}\frac{du_e}{d\xi}\frac{T_o}{T_e}\,t_s = \Phi_1(\eta) \tag{33}$$

If we assume the relationship between u_e and ξ to be given by

$$u_e \sim \sqrt{T_e/T_o}\;\xi^m \tag{34a}$$

then

$$\frac{2\xi}{u_e}\frac{du_e}{d\xi}\left(\frac{T_o}{T_e}\right)t_s = 2m \equiv \beta \tag{34b}$$

where β is the modified Falkner-Skan parameter.

Using Eq. (34) to define the parameter β, the equations assume the following similarity form:

$$\left(\lambda\frac{r^{2j}}{r_k^{2j}}f''\right)' + ff'' = \beta\left\{f'^2 - \frac{1}{t_s}\left[(1 - t_w)\theta - (1 - t_s)g^2 + t_w\right]\right\} \tag{35}$$

$$\left(\lambda\frac{r^{2j}}{r_k^{2j}}g'\right)' + fg' = 0 \tag{36}$$

$$\left(\lambda\frac{r^{2j}}{r_k^{2j}}\frac{G'}{Pr}\right)' + fG' = \left\{\left[\lambda\frac{r^{2j}}{r_k^{2j}}\left(\frac{1}{Pr} - 1\right)\right]2\sigma\left[f''f'\left(\frac{u_e}{u_\infty}\right)^2\cos^2\Lambda + gg'\sin^2\Lambda\right]\right\}'$$

$$+ \frac{c_{1w}}{H_e}\left[\lambda\frac{r^{2j}}{r_k^{2j}}\frac{1}{Sc}\left(\frac{1}{Le} - 1\right)(h_1 - h_2)Z_1'\right]' \tag{37}$$

$$\left(\lambda \frac{r^{2j}}{r_k^{2j}} \frac{1}{Sc} Z_1'\right)' + fZ_1' = 0 \tag{38}$$

In Eq. (37), $\sigma = (U_\infty^2/2H_e)$ and is given the designation of hypersonic parameter. Two modified hypersonic parameters can be defined, as follows

$$\sigma_1 = \left(\frac{U_\infty^2}{2H_e}\right)\left(\frac{u_e}{u_\infty}\right)^2 \tag{39a}$$

and

$$\sigma_2 = \left(\frac{U_\infty^2}{2H_e}\right)\left[\left(\frac{u_e}{u_\infty}\right)^2 \cos^2 \Lambda + \sin^2 \Lambda\right] \tag{39b}$$

The parameter σ_1 includes the shock-angle effect on the energy equation; it is σ times the ratio of the velocity at the edge of the boundary layer to the free-stream velocity. The parameter σ_2 is a generalization of σ_1 including the effect of sweep angle. When $\sigma = 0$, the ratio of kinetic energy in the flow to the stagnation enthalpy of the free stream is zero; as a result the dissipation term in the energy equation is neglected. As σ increases, the dissipation term plays an ever-increasing role in the energy equation.

EQUATIONS FOR SOLUTION

Subject to simplification regarding the flows to be considered, Eqs. (35) to (38) are those equations for which solutions are presented in this Chapter. These simplifications are:

1. The Chapman-Rubesin constant is defined as

$$C = \rho_w \mu_w / \rho_e \mu_e$$

so that the quantities ξ and λ become

$$\xi = \int_0^x r_k^{2j} \rho_w \mu_w u_e \, dx$$

$$\lambda = \rho\mu / \rho_w \mu_w$$

2. Curvature effects are neglected:

$$(r/r_k)^2 = 1$$

3. The product $(Le - 1) \cdot (h_2 - h_1)$ is assumed to be zero, and thermal diffusion is neglected. A heuristic justification of this simplification for equilibrium air is given by Beckwith and Cohen, Appendix A of Ref. 6.

The Lewis number has been taken to be unity; i.e., Pr = Sc. As a final step, the concentration variable Z_1 is transformed to the dependent variable

$$Z = 1 - Z_1 = \frac{c_{1w} - c_1}{c_{1w}}$$

Substitution of θ for G and z for Z_1 yields the equations that were solved exactly to prepare the tables of solutions:

$$(\lambda f'')' + ff'' = \beta\left\{f'^2 - \frac{1}{t_s}\left[(1 - t_w)\theta - (1 - t_s)g^2 + t_w\right]\right\} \quad (40)$$

$$(\lambda g')' + fg' = 0 \tag{41}$$

$$\left(\lambda \frac{\theta'}{Pr}\right)' + f\theta' = \left\{ \frac{2\sigma\lambda}{(1 - t_w)} \left(\frac{1}{Pr} - 1\right) \left[f''f'\left(\frac{u_e}{u_\infty}\right)^2 \cos^2 \Lambda + gg' \sin^2 \Lambda \right] \right\}' \tag{42}$$

$$\left(\lambda \frac{z'}{Sc}\right)' + fz' = 0 \tag{43}$$

This is a 9^{th}-order nonlinear set of ordinary differential equations. Nine boundary conditions are required, five at the wall and four at the edge of the boundary layer. The boundary conditions to be satisfied are:

At the wall when $\eta = 0$:

$$f'(0) = g(0) = \theta(0) = z(0) = 0 \tag{44}$$

and the Eckert-Schneider condition which relates the conservation of 2-particles at the wall:

$$f(0) = - \frac{c_{1w}}{Sc(1 - c_{1w})} z'(0) \tag{45}$$

In obtaining Eq. (45) from Eq. (22a), we have made use of the fact that $(\lambda)_w = 1$. At the outer edge of the boundary layer, the conditions are:

$$f'(\eta_e) = g(\eta_e) = \theta(\eta_e) = z(\eta_e) = 1 \tag{46}$$

The value of η_e is chosen to be large enough to insure that the boundary conditions given by Eq. (46) are satisfied asymptotically to a high degree of accuracy (see Section IV).

The similarity conditions are satisfied by:

$$\beta = \text{constant} \tag{47}$$

$$\lambda = \rho\mu/\rho_w\mu_w = \lambda(\eta) \tag{48}$$

$$\left.\begin{aligned} \text{either } Pr &= 1 \\ \text{or } (u_e/u_\infty)^2 &= \text{constant and } Pr = Pr(\eta) \end{aligned}\right\} \tag{49}$$

$$Sc = Sc(\eta) \; [= Pr(\eta) \text{ for } Le = 1] \tag{50}$$

$$t_w \text{ and } c_w \text{ are constants} \tag{51}$$

$$f(0) = \text{constant} \equiv f_w \tag{52}$$

The viscosity-temperature relationship may be characterized by the power-law expression: $\mu = AT^{\omega}$. This yields $\lambda \sim T^{\omega-1}$. A value $\omega = 0.7$ corresponds to conventional wind-tunnel conditions, while $\omega = 0.5$ represents conditions encountered in hypersonic flight.

In terms of the similarity variables, λ may be written in the form

$$\lambda = \left|\frac{1}{t_w}\left\{(1 - t_w)\theta - (1 - t_s)g^2 - \sigma_1 (\cos^2 \Lambda)f'^2 + t_w\right\}\right|^{\omega-1}$$

If $\omega = 1$, then $\lambda = 1$ and $\rho\mu = $ constant. In the literature, this latter assumption has often been made in order to simplify the boundary-layer equations.

The authors have demonstrated in a previous paper(4) that boundary-layer characteristics calculated by means of a Sutherland viscosity

law can be approximated almost exactly using the power-law relation $\mu \sim T^{\omega_r}$, provided the empirical exponent ω_r is suitably chosen. Sutherland's law may be written

$$\frac{\mu}{\mu_o} = t^{3/2} \left[\frac{s + 1}{s + t} \right] \tag{53a}$$

where $t = t/T_o$ and $s = S/T_o$. The Sutherland constant, S, is a characteristic temperature for the gas, and μ_o is the viscosity evaluated at the stagnation temperature T_o. The empirical equations for calculating ω_r are

$$\omega_r = \frac{3}{2} + \frac{\ln \dfrac{t_w + s}{t_r + s}}{\ln t_r} \tag{53b}$$

$$t_r = 0.5\,(t_e + t_w) + 0.22\,(1 - t_w) \tag{53c}$$

The quantity t_r is sometimes referred to as the Eckert reference temperature.

Solution of Eqs. (40) to (43) subject to the boundary conditions of Eqs. (44) to (46) requires the specification of eight independent parameters. These are

$f_w = f(0)$ = blowing parameter

Pr = Prandtl number, taken to be constant = Sc

t_s = sweep parameter

t_w = normalized wall temperature

$(u_e/u_\infty)^2$ = local streamwise velocity ratio

β = pressure-gradient parameter

$\sigma \equiv U_\infty^2/2H_e$ = hypersonic parameter

ω or s = temperature-viscosity law

All eight must be independent of the streamwise coordinate ξ for similarity to hold exactly. If Pr = 1, the parameters σ and $(u_e/u_\infty)^2$ are not required.

It should be emphasized that thermal diffusion has been neglected and the product $(Le - 1)(h_1 - h_2)$ has been assumed zero. In this approximation, solution of the three equations for f, θ, and g are independent of the diffusion equation and, consequently, are independent of the value of the Schmidt number.

SPECIAL CLASSES OF REDUCED EQUATIONS

Two-Dimensional Flow

If $r^{2j}/r_k^{2j} = 1$, the ordinary two-dimensional boundary-layer equations result:

$$(\lambda f'')' + ff'' = \beta\left\{f'^2 - \frac{1}{t_s}\left[(1 - t_w)\theta - (1 - t_s)g^2 + t_w\right]\right\} \tag{54}$$

$$(\lambda g')' + fg' = 0 \tag{55}$$

$$\left(\lambda \frac{\theta'}{Pr}\right)' + f\theta' = \left\{\frac{2\sigma\lambda}{(1 - t_w)}\left(\frac{1}{Pr} - 1\right)\left[f''f'\left(\frac{u_e^2}{u_\infty^2}\right)\cos^2\Lambda + gg'\sin^2\Lambda\right]\right\}'$$

$$+ \frac{c_{1w}}{H_e}\left[\frac{\lambda}{Sc}\left(\frac{1}{Le} - 1\right)(h_1 - h_2)Z_1'\right]' \tag{56}$$

$$\left(\lambda \frac{Z_1'}{Sc}\right)' + fZ_1' = 0 \tag{57}$$

Equations (54) to (57) are valid for axisymmetric flow if Λ is set equal to zero and j is unity.

$\underline{\Lambda = 0}$

When the free-stream-flow direction coincides with the x-direction, then $\Lambda = 0$ and $t_s \equiv 1$. For this case, the boundary-layer equations reduce to:

$$(\lambda f'')' + ff'' = \beta[f'^2 - (1 - t_w)\theta + t_w] \tag{58}$$

$$\left(\frac{\theta'}{Pr}\right)' + f\theta' = \left\{\left(\frac{1}{Pr} - 1\right)\frac{2\sigma\lambda}{(1 - t_w)}\left(\frac{u_e^2}{u_\infty^2}\right)f''f'\right\}'$$

$$+ \frac{c_{1w}}{H_e}\left[\frac{\lambda}{Sc}\left(\frac{1}{Le} - 1\right)(h_1 - h_2)Z_1'\right]' \tag{59}$$

$$\left(\lambda \frac{Z_1'}{Sc}\right)' + fZ_1' = 0 \tag{60}$$

<u>No Mass Transfer</u>

In the special case where no mass transfer takes place in the boundary layer, $Z_1 \equiv 0$ and $c_{1w} \equiv 0$, and the boundary conditions (including the Eckert-Schneider condition) are ignored:*

---

*Although the solution for the concentration ratio Z_1 is $Z_1 = 0$, the solution of Eq. (43) for the normalized concentration z has a nonvanishing solution and a nonzero gradient z' at the wall.

$$(\lambda f'')' + ff'' = \beta\{f'^2 - (1 - t_w)\theta - t_w\} \qquad (61)$$

$$\left(\lambda \frac{\theta'}{Pr}\right)' + f\theta' = \left[\left(\frac{1}{Pr} - 1\right) \frac{2\sigma\lambda}{(1 - t_w)} f''f' \left(\frac{u_e^2}{u_\infty^2}\right)\right]' \qquad (62)$$

Equations (61) and (62) are the ordinary two-dimensional Prandtl boundary-layer equation.

$\underline{\mu \sim T}$

If the viscosity is directly proportional to the temperature, $\lambda \equiv 1$, and Eqs. (61) and (62) reduce to:

$$f''' + ff'' = \beta\left\{f'^2 - (1 - t_w)\theta - t_w\right\} \qquad (63)$$

$$\left(\frac{\theta'}{Pr}\right)' + f\theta' = \left[\left(\frac{1}{Pr} - 1\right)\frac{2\sigma f''f'}{(1 - t_w)}\left(\frac{u_e^2}{u_\infty^2}\right)\right]' \qquad (64)$$

$\underline{Pr = 1}$

Equations (63) and (64) reduce to a form in which the dissipation term of the energy equation is zero:

$$f''' + ff'' = \beta\left[f'^2 - (1 - t_w)\theta - t_w\right] \qquad (65)$$

$$\theta'' + f\theta' = 0 \qquad (66)$$

$\underline{\beta = 0}$

Finally, if the pressure-gradient parameter is zero, the momentum equation is uncoupled from the energy equation; Eq. (65) becomes

$$f''' + ff'' = 0 \tag{67}$$

and by inspection, the solution of the energy equation is

$$\theta = f' \tag{68}$$

Computation of Boundary-Layer Properties

Heat-transfer rates and skin friction are related by the similarity transformations to the derivatives $\theta'(0)$, $f''(0)$, and $g'(0)$. The heat transfer from the stream to the wall is given by

$$q_w = + k_w(\partial T/\partial y)_w \tag{69a}$$

After the proper substitutions have been made, the expression for the heat transfer in similarity coordinates is

$$q_w = + \frac{k_w \rho_w u_e H_e (1 - t_w) r_k^j}{C_{p_w} \sqrt{2\xi}} \theta'(0) \tag{69b}$$

and the local heat transfer coefficient is

$$C_h = q_w / [\rho_\infty U_\infty (H_{aw} - H_w)] \tag{70}$$

The skin friction for the x-direction can be described in similarity coordinates as follows:

$$\tau_x = \mu_w \frac{r_k^j \rho_w u_e^2}{\sqrt{2\xi}} f''(0) \tag{71}$$

The spanwise coefficient is

$$\tau_z = \mu_w \frac{r_k^j \rho_w u_e w_e}{\sqrt{2\xi}} g'(0) \tag{72}$$

The total component of skin friction is the vector sum of the two components τ_x and τ_z:

$$\tau = \mu_w \frac{r_k^j \rho_w u_e}{\sqrt{2\xi}} \left[\{f''(0)\}^2 u_e^2 + \{g'(0)\}^2 w_e^2\right]^{1/2} \tag{73}$$

and the local skin-friction coefficient is defined by

$$C_f = 2\tau/\rho_\infty U_\infty^2 \tag{74}$$

Several integral relations have also been tabulated. They are

$$I_1 = (1 - t_s)[I_1(1) - I_1(2)] - (1 - t_w)I_1(3) + I_1(2) \tag{75}$$

$$I_2 = \int_0^\infty f'(1 - f')\, d\eta \tag{76}$$

where

$$I_1(1) = \int_0^\infty (1 - g^2)\, d\eta \tag{77}$$

$$I_1(2) = \int_0^\infty (1 - f'^2)\, d\eta \tag{78}$$

$$I_1(3) = \int_0^\infty (1 - \theta)\, d\eta \tag{79}$$

These integrals can be related to the boundary-layer-thickness parameters expressed in similarity coordinates.

Displacement thickness:

$$\delta^* = \int_0^\infty \left(1 - \frac{\rho u}{\rho_e u_e}\right) dy = \frac{\sqrt{2\xi}}{\rho_e u_e r_k^j}\left(\frac{T_o}{T_e}\right)\left[I_1 - \left(\frac{T_e}{T_o}\right) I_2\right] \tag{80}$$

Momentum thickness:

$$\Theta = \int_0^\infty \frac{\rho u}{\rho_e u_e}\left(1 - \frac{u}{u_e}\right) dy = \frac{\sqrt{2\xi}}{\rho_e u_e r_k^j} I_2 \tag{81}$$

RANGE OF SOLUTIONS AND PARAMETERS

The parameters for the systems of equations under consideration here are

pressure-gradient parameter	$0 \le \beta \le 5$ and $\beta = \infty$
Prandtl number	$Pr = 0.5, 0.7, 1.0$
Schmidt number	$Sc = Pr$ (i.e., $Le = 1$)
temperature-viscosity-law parameter	$\omega = 0.5, 0.7, 1.0$ or $.01 \le s \le .3$
sweep-angle parameter	$0 \le t_s \le 1$
mass-transfer parameter	$-0.6 \le f_w \le 0$
wall temperature	$0 \le t_w \le 1.2$
shock-angle parameter	$(u_e/u_\infty)^2 = 0, 0.5, 1.0$
hypersonic parameter	$\sigma = U_\infty^2/2H_e = 0, 0.5, 1.0$

The quantities tabulated are

f_w'' = normalized chordwise velocity gradient at wall, $[f''(0)]$

θ_w' = normalized total enthalpy gradient at wall, $[\theta'(0)]$

g_w' = normalized transverse velocity gradient at wall, $[g'(0)]$

plus the integrals I_1, I_2, $I_1(1)$, $I_1(2)$, $I_1(3)$ as defined by Eqs. (75) to (79).

III. NUMERICAL RESULTS

This section provides a guide to the present solutions of the laminar boundary-layer equations. The available solutions are shown schematically in Keys 1 to 6. Each key shows all of the values of the eight independent parameters covered by the correspondingly numbered table found at the end of this Memorandum. If the key indicates (by an x in the parameter matrix) that a solution is available, then the numerical values are given in the correspondingly numbered table. The key also provides the reader with a graphic display of the solutions available in the neighborhood of his range of interest. Inner solutions for the limit $\beta \to \infty$ are given in Table 7 which is too short to warrant a separate key.

Approximately a sixth of the solutions listed have been reported by other authors. All values listed have been computed, or recomputed, according to the numerical procedures described in Section IV. Some differences exist between the present values and those of earlier investigators, but the present values are believed to be accurate within the limits prescribed in the succeeding three sections (generally, correct to four significant figures).

The most extensive of the earlier tabulations of solutions to the similarity equations for a laminar boundary layer of a perfect gas may be found in the works of Beckwith,[3] Beckwith and Cohen,[6] Li and Nagamatsu,[7] Cohen and Reshotko,[8] Reshotko and Beckwith,[9] and Dewey.[10] Solutions of high accuracy have been obtained by Smith[11,12] and others. Early contributions to the understanding of the role of fluid properties may be found in the works of Busemann,[13] Karman and

Tsien,[14] Crocco,[15] Young and Janssen,[16] and van Driest.[17] Emmons and Leigh[18] report a large number of solutions with the surface mass-transfer parameter f_w other than zero. Several recent compendiums[19 to 22] on boundary-layer theory may be consulted for a more complete description of previous contributions.

Since this Memorandum is restricted to positive values of the pressure-gradient parameter β and values $f_w \leq 0$ corresponding to mass injection into the boundary layer, a list of sources is offered in which solutions for $\beta < 0$ and $f_w > 0$ may be found. Emmons and Leigh[18] have obtained solutions for $\beta = 0$ and values of $f_w\sqrt{2}$ equal to 10, 6, , 4, 3, 2.5, 2, 1.5, 1.4, 1.3, ...5, .45, .40, ... 0.1, and 0.05, assuming that Pr = 1 and the viscosity-temperature law is linear. Suction results are reported also by Spalding and Evans,[23] Pretsch,[24] Watson,[25] Eckert, Donoughe, and Moore,[26] Thwaites,[27] Schlichting and Bussman,[28] Mangler,[29] Schaefer,[30] Emmons and Leigh,[18] and Koh and Hartnett.[31]

Negative values of the pressure-gradient parameter β have been considered by Hartree,[32] Stewartson,[21,33] Smith,[11,12] Cohen and Reshotko,[8] Beckwith,[3] and Hufen and Wuest.[34] Solutions for negative values of β differ from their counterparts for positive β in two ways. First, two sets of "proper" solutions exist for each value of β; second, every value of the wall derivative f''_w and its corresponding value of θ'_w, etc., will satisfy the boundary conditions $f' = 0$ at $\eta = 0$ and $f' \to 1$ as $\eta \to \infty$. The "proper" solution must, therefore, be defined as that solution for which f' approaches unity most rapidly from above (see Refs. 3, 8, and 21).

Key 1

Similar Solutions for $\omega = Pr = 1$

			t_w								
β	f_w	t_s	0	.15	.2	.4	.5	.6	.8	1.0	2.0
0.0[a]	0.0	all	x							x	
	-0.1414	"	x							x	
	-0.2828	"	x							x	
	-0.4243	"	x							x	
	-0.5657	"	x							x	
	-0.7071	"	x							x	
	-0.7782	"	x							x	
	-0.8485	"	x							x	
	-0.8755	"	x							x	
0.05	0.0	1.0	x				x			x	
0.1	0.0	1.0		x		x		x		x	
0.2	0.0	0.1539	x				x			x	
		0.3333	x				x			x	
		0.6250	x				x			x	
		1.0	x				x			x	
0.25	0.0	0.1000	x				x			x	
0.2857	0.0	1.0	x		x	x		x	x	x	
	-0.2	1.0	x		x	x		x	x	x	
	-0.4	1.0	x		x	x		x	x	x	
	-0.6	1.0	x		x	x		x	x	x	
0.3	0.0	1.0	x		x	x		x	x	x	x
0.4	0.0	1.0	x		x	x		x	x	x	
	-0.2	1.0	x		x	x		x	x	x	
	-0.4	1.0	x			x		x	x	x	
	-0.6	1.0	x		x	x		x	x	x	
0.5	0.0	0.1000	x				x			x	
		0.1539	x				x			x	
		0.3333	x				x			x	
		0.6250	x				x			x	
		1.0	x	x	x	x	x	x	x	x	x
	-0.5	0.1539	x				x			x	
		0.3333	x				x			x	
		0.6250	x				x			x	
		1.0	x				x			x	
0.75	0.0	0.1000	x				x			x	
		0.3333	x				x			x	
		1.0	x	x		x	x	x		x	

[a] All solutions for $Pr = \omega = 1$ and $\beta = 0$ are linear in t_w.

Key 1, cont.

β	f_w	t_s	t_w: 0	.15	.2	.4	.5	.6	.8	1.0	2.0
1.0	0.0	0.1000	x				x			x	
		0.1539	x				x			x	x
		0.2500	x				x			x	
		0.3333	x				x			x	
		0.5000	x				x			x	x
		0.6250	x				x			x	
		1.0	x	x	x	x	x	x	x	x	x
	-0.5	0.1539	x				x			x	
		0.3333	x				x			x	
		0.6250	x				x			x	
		0.8333	x				x			x	
		1.0	x				x			x	
	-1.0	0.1539	x				x			x	
		0.3333	x				x			x	
		0.6250	x				x			x	
		1.0	x				x			x	
1.4	0.0	0.1000	x				x			x	
1.5	0.0	0.1539	x				x			x	
		0.3333	x				x			x	
		0.6250	x				x			x	
		1.0	x	x	x	x	x	x	x	x	x
1.8	0.0	0.1000	x				x			x	
2.0	0.0	0.1000	x				x			x	
		0.1539	x				x			x	
		0.3333	x				x			x	
		0.6250	x				x			x	
		1.0	x	x	x	x	x	x	x	x	x
2.4	0.0	0.1539	x				x			x	
		0.3333	x				x			x	
		0.6250	x				x			x	
		1.0	x				x			x	
2.8	0.0	0.1000	x				x			x	
		0.1539	x							x	
		0.3333	x							x	
		0.6250	x				x			x	
		1.0	x				x			x	
3.4	0.0	1.0					x			x	
4.0	0.0	1.0	x				x			x	
5.0	0.0	1.0	x				x			x	

Key 2

Similar Solutions for a Power-Law Viscosity Relation
$Pr = 0.7,\ f_w = 0$

β	ω	t_s	σ	$\left(\frac{u_e}{u_\infty}\right)^2$	t_w .05	.1	.15	.4	.6	1.0	1.1	t_{aw}
0.0	0.5	all	0.0	...			x	x	x	x		
			0.5	...			x	x	x			x
			1.0	...	x		x	x	x		x	x
	0.7	all	0.0	...			x	x	x	x		
			0.5	...			x	x	x			x
			1.0	...	x		x	x	x		x	x
	1.0	all	0.0	...			x	x	x	x		
			0.5	...			x	x	x			x
			1.0	...			x	x	x			x
0.1	0.5	1.0	1.0	...	x		x	x	x		x	x
	0.7	0.3333	1.0	1.0			x					x
		0.5000	1.0	1.0			x					x
		0.6250	1.0	1.0			x					x
		1.0	1.0	...	x		x	x	x		x	x
	1.0	1.0	1.0	...			x	x	x			x
0.2	0.5	0.3333	1.0	1.0			x					x
		0.6250	1.0	1.0			x					x
		1.0	1.0	...	x		x	x	x		x	x
	0.7	0.3333	1.0	1.0			x		x			x
		0.5000	1.0	1.0			x					x
		0.6250	1.0	1.0			x		x			x
		1.0	1.0	...	x		x	x	x		x	x
	1.0	0.3333	1.0	1.0			x					x
		0.6250	1.0	1.0			x					x
		1.0	1.0	...			x	x				x
0.2857	0.5	0.3333	1.0	1.0			x					x
		0.6250	1.0	1.0			x					x
		1.0	0.0	...			x			x		
			0.5	...			x					x
			1.0	...	x		x	x	x		x	x
	0.7	0.3333	1.0	1.0			x					x
		0.5000	1.0	1.0			x					x
		0.6250	1.0	1.0			x					x
		1.0	1.0	...	x		x	x	x		x	x
	1.0	1.0	1.0	...			x	x	x			x

Key 2, cont.

β	ω	t_s	σ	$\left(\frac{u_e}{u_\infty}\right)^2$	t_w							t_{aw}
					.05	.1	.15	.4	.6	1.0	1.1	
0.4	0.5	0.3333	1.0	0.0			x					x
				0.5			x					x
				1.0			x					x
		0.6250	1.0	0.0			x					x
				0.5			x					x
				1.0			x					x
		1.0	0.0	...			x			x		
			0.5	...			x					x
			1.0	...	x		x	x	x		x	x
	0.7	0.3333	1.0	0.0			x					x
				0.5			x					x
				1.0			x					x
		0.6250	1.0	0.0			x					x
				0.5			x					x
				1.0			x					x
		1.0	0.0	...			x			x		
			0.5	...			x	x	x			x
			1.0	...	x		x	x	x		x	x
	1.0	0.3333	1.0	1.0			x					x
		0.6250	1.0	1.0			x					x
		1.0	0.0	...			x			x		
			0.5	...			x					x
			1.0	...			x	x	x			x
0.5	0.5	0.3333	1.0	0.5			x					x
		1.0	0.0	...	x		x	x	x	x	x	
			0.25	...	x		x					x
			0.5	...	x		x	x	x		x	x
			0.6	...	x		x					x
			0.8	...	x	x	x					x
			1.0	...	x		x	x	x		x	x
	0.7	1.0	0.0	...	x		x	x	x	x	x	
			0.5	...	x		x	x	x		x	x
			1.0	...	x		x	x	x		x	x
	1.0	1.0	0.0	...			x	x	x	x		
			1.0	...			x	x	x		x	x
0.75	0.5	1.0	0.5	...	x		x	x	x		x	x
	0.7	1.0	0.0	...	x		x	x	x	x	x	
			0.5	...	x		x	x	x		x	x

Key 2, cont.

β	ω	t_s	σ	$\left(\frac{u_e}{u_\infty}\right)^2$	t_w .05	.1	.15	.4	.6	1.0	1.1	t_{aw}
1.0	0.5	0.3333	1.0	0.0	x		x					x
		0.6250	1.0	0.0	x		x					x
		1.0	0.0	...	x		x	x	x	x	x	
			0.25	...	x		x					x
			0.5	...	x		x	x	x		x	x
			0.6	...	x	x	x					x
			0.8	...	x	x	x					x
			1.0	...	x		x	x	x		x	x
	0.7	0.3333	1.0	0.0	x		x					x
		0.6250	1.0	0.0	x		x					x
		1.0	0.0	...	x		x	x	x	x	x	
			0.5	...	x		x	x	x		x	x
			0.9	...			x	x	x			x
			1.0	...	x		x	x	x		x	x
	1.0	0.1539	1.0	0.0			x					x
		0.3333	1.0	0.0			x					x
		0.6250	1.0	0.0			x					x
		1.0	0.0	...			x	x		x		
1.4	0.7	1.0	0.6	...			x	x	x		x	x
1.5	0.5	1.0	0.0	...	x		x	x	x	x		
			0.25	...	x		x					x
			0.5	...	x		x	x	x			x
	0.7	1.0	0.0	...	x				x	x		
1.8	0.5	1.0	0.6	...		x	x	x	x		x	
2.0	0.5	1.0	0.0	...	x		x	x	x	x		
			0.25	...	x		x					x
	0.7	1.0	0.0	...	x		x	x	x	x		
3.0	0.5	1.0	0.0	...			x	x	x	x		
	0.7	1.0	0.0	...	x		x	x	x	x		

Key 3

Similar Solutions for Pr = 0.7, t_s = 1, $f_w \neq 0$

				t_w					
β	ω	f_w	σ_1	.5	.15	.4	.6	1.0	t_{aw}
0.0	0.5	0.0	1.0		x				x
		-0.2	1.0		x				x
		-0.4	1.0		x				x
		-0.6	1.0		x	x	x		
	0.7	0.0	0.0		x			x	
			0.5		x				x
			1.0		x				x
		-0.2	0.0		x			x	
			0.5		x				x
			1.0		x				x
		-0.4	0.0		x			x	
			0.5		x				x
			1.0		x				x
		-0.6	0.0		x	x	x	x	
			0.5		x				x
			1.0		x	x	x		x
0.2	0.5	0.0	1.0		x				x
		-0.2	1.0		x				x
		-0.4	1.0		x				x
		-0.6	1.0		x	x	x		x
	0.7	0.0	1.0		x				x
		-0.2	1.0		x				x
		-0.4	1.0		x				x
		-0.6	1.0		x	x	x		x
0.4	0.5	0.0	1.0		x				x
		-0.2	1.0		x				x
		-0.4	1.0		x				x
		-0.6	1.0		x				x
	0.7	0.0	0.0		x			x	
			0.5		x				x
			1.0		x				x
		-0.2	0.0		x				x
			0.5		x				x
			1.0		x				
		-0.4	0.0		x			x	
			0.5		x				x
			1.0		x				x

Key 3, cont.

β	ω	f_w	σ_1	t_w .5	.15	.4	.6	1.0	t_{aw}
0.4	0.7	-0.6	0.0		x			x	
			0.5			x	x		x
			1.0		x	x	x		x
0.5	0.7	0.0	0.0		x			x	
		-0.2	0.0		x			x	
		-0.4	0.0		x			x	
		-0.6	0.0		x	x	x	x	
		-0.8	0.0	x				x	
		-1.0	0.0		x			x	
		-1.2	0.0		x	x	x	x	
		-1.4	0.0					x	
0.75	0.7	0.0	0.5		x				x
		-0.2	0.5		x				x
		-0.4	0.5		x				x
		-0.6	0.5		x	x	x		x
1.0	0.5	0.0	0.0		x			x	
		-0.2	0.0		x			x	
		-0.4	0.0		x			x	
		-0.6	0.0			x	x	x	
	0.7	0.0	0.0		x			x	
		-0.2	0.0		x			x	
		-0.4	0.0		x			x	
		-0.6	0.0		x	x	x	x	
		-0.8	0.0					x	
		-1.2	0.0					x	
		-1.4	0.0					x	
		-1.6	0.0					x	
	1.0	0.0	0.0		x			x	
		-0.2	0.0		x			x	
		-0.4	0.0		x			x	
		-0.6	0.0		x			x	
		-0.8	0.0		x			x	
		-1.0	0.0		x				
		-1.2	0.0		x			x	
		-1.4	0.0		x			x	
		-1.5	0.0		x			x	
		-1.6	0.0		x			x	

Key 3, cont.

β	ω	f_w	σ_1	t_w .5	.15	.4	.6	1.0	t_{aw}
2.0	0.5	0.0	0.0			x	x	x	
		-0.4	0.0				x	x	
	0.7	-0.2	0.0				x	x	
		-0.4	0.0			x		x	
			1.0			x	x		
		-0.6	0.0		x	x	x	x	
5.0	0.5	-0.2	1.0				x		
		-0.6	1.0			x			

Key 4

Similar Solutions for $t_s = 1$, $f_w = 0$

β	ω	σ_1	Pr	t_w: 0	.05	.1	.15	.2	.4	.5	.6	.8	1.0	1.1	2.0	t_{aw}
0.0	0.5	0.0	0.7				x		x		x		x			
		0.5	0.5				x		x		x					x
			0.7				x		x		x					x
		1.0	0.5				x		x		x					
			0.7		x		x		x		x			x		x
			1.0				x		x		x		x	x		
	0.7	0.0	0.7				x		x		x		x			
			1.0								x		x			
		0.5	0.7				x		x		x					x
		1.0	0.5				x		x		x					x
			0.7		x		x		x		x			x		x
			1.0				x									
	1.0	0.0	0.7				x		x		x		x			
			1.0	x									x			
		0.5	0.5				x		x		x					x
			0.7				x		x		x					x
		1.0	0.5				x		x		x					
			0.7				x		x		x					x
			1.0				x									
0.05	1.0	0.0	1.0	x						x			x			
0.10	0.5	1.0	0.7		x		x		x		x					x
	0.7	1.0	0.7		x		x		x		x			x		x
	1.0	0.0	1.0				x		x		x		x			x
		1.0	0.7				x		x		x					x
0.15	0.7	0.0	0.7								x					
0.2	0.5	1.0	0.7		x		x		x		x			x		x
	0.7	1.0	0.7		x		x		x		x			x		x
	1.0	0.0	1.0	x						x			x			
		1.0	0.7				x		x							x
0.2857	0.5	0.0	0.7				x						x			
		0.5	0.7				x									x
		1.0	0.7		x		x		x		x			x		x
	0.7	1.0	0.7		x		x		x		x			x		x
	1.0	0.0	1.0	x				x	x		x	x	x			
		1.0	0.7				x		x		x					x
0.3	1.0	0.0	1.0	x				x	x		x	x	x		x	

Key 4, cont.

				t_w												
β	ω	σ_1	Pr	0	.05	.1	.15	.2	.4	.5	.6	.8	1.0	1.1	2.0	t_{aw}
0.4	0.5	0.0	0.7				x						x			
		0.5	0.7				x									x
		1.0	0.7		x		x		x		x			x		x
	0.7	0.0	0.7				x						x			
		0.5	0.7				x		x		x					x
		1.0	0.7		x		x		x		x			x		x
	1.0	0.0	0.7				x						x			
			1.0	x				x	x		x	x	x			
		0.5	0.7				x									x
		1.0	0.7				x		x		x					x
0.5	0.5	0.0	0.5				x		x		x					
			0.7		x		x		x		x		x	x		
			1.0				x		x		x					
		0.25	0.7		x		x									x
		0.5	0.7		x		x		x		x			x		x
		0.6	0.7		x		x									x
		0.8	0.7		x	x	x									x
		1.0	0.7		x		x		x		x			x		x
	0.7	0.0	0.5				x									
			0.7		x		x		x		x		x	x		
			1.0						x		x		x			
		0.5	0.7		x		x		x		x			x		x
		1.0	0.5				x		x		x					
			0.7		x		x		x		x			x		x
	1.0	0.0	0.5				x		x		x		x			
			0.7				x		x		x		x			
			1.0	x		x	x	x	x	x	x	x	x		x	
		1.0	0.5				x		x		x					x
			0.7				x		x		x			x		x
0.75	0.5	0.5	0.7		x		x		x		x			x		x
	0.7	0.0	0.7		x		x		x		x		x	x		
		0.5	0.7		x		x		x		x			x		x
	1.0	0.0	1.0	x			x		x	x	x		x			
1.0	0.5	0.0	0.5				x		x		x		x			
			0.7		x		x		x		x		x	x		
			1.0				x		x		x		x			
		0.25	0.7		x		x									x
		0.5	0.7		x		x		x		x			x		x
		0.6	0.7		x	x	x									x
		0.8	0.7		x	x	x									x
		1.0	0.7		x		x		x		x			x		x

Key 4, cont.

				t_w												
β	ω	σ_1	Pr	0	.05	.1	.15	.2	.4	.5	.6	.8	1.0	1.1	2.0	t_{aw}
1.0	0.7	0.0	0.5			x			x		x					
			0.7		x		x		x		x		x	x		
			1.0				x				x		x			
		0.5	0.7		x		x		x		x			x		x
		0.9	0.7				x		x		x					x
		1.0	0.7		x		x		x		x			x		x
	1.0	0.0	0.5						x		x		x			
			0.7	x			x		x	x	x		x			
			1.0	x			x	x	x	x	x	x	x		x	
1.4	0.5	0.6	0.7			x										
		0.8	0.7			x										
	0.7	0.6	0.7				x		x		x			x		x
1.5	0.5	0.0	0.7		x		x		x		x		x			
		0.25	0.7		x		x									x
		0.5	0.7		x		x		x		x					x
	0.7	0.0	0.7		x		x		x				x			
	1.0	0.0	1.0	x			x	x	x	x	x	x	x		x	
1.8	0.5	0.6	0.7			x	x		x		x			x		
		0.8	0.7			x										
2.0	0.5	0.0	0.5				x		x		x					
			0.7		x		x		x		x		x			
		0.25	0.7		x		x									x
	0.7	0.0	0.5				x		x		x		x			
			0.7		x		x		x		x		x			
		0.5	0.5				x		x		x					
	1.0	0.0	1.0	x			x	x	x	x	x	x	x		x	
2.4	1.0	0.0	1.0	x						x			x			
2.8	1.0	0.0	1.0	x						x			x			
3.0	0.5	0.0	0.7				x		x		x		x			
	0.7	0.0	0.7		x		x		x		x		x			
3.4	1.0	0.0	1.0	x						x			x			
4.0	1.0	0.0	1.0	x						x			x			
5.0	1.0	0.0	1.0	x						x			x			

Key 5[a]

Similar Solutions for a Sutherland Viscosity--Temperature Relation,
Pr = 0.7, t_s = 1.0

				t_w										
ξ	s	f_w	σ_1	.05	.15	.2	.4	.5	.6	.7	.8	1.0	1.1	t_{aw}
0.0	0.01	0.0	0.0		x		x		x			(b)		
			0.5		x				x					x
			1.0		x				x	x				
		-0.2	0.0		x		x		x			(b)		
		-0.6	0.0		x		x		x			(b)		
	0.03	0.0	1.0				x							
	0.05	0.0	1.0		x	x	x		x					
	0.1	0.0	0.0		x		x		x			(b)		
			0.5		x		x							
	0.3	0.0	0.0		x		x		x					
			0.5		x		x		x					x
			1.0		x				x					
		-0.2	0.0		x		x		x			(b)		
		-0.6	0.0		x		x		x			(b)		
0.4	0.05	0.0	1.0		x		x						x	
0.5	0.01	0.0	0.0		x		x		x			(b)		
		-0.2	0.0		x		x		x			(b)		
		-0.6	0.0		x		x		x			(b)		
	0.02	0.0	0.0			x								
	0.3	0.0	0.0				x		x			(b)		
		-0.2	0.0		x		x		x			(b)		
		-0.6	0.0		x		x		x			(b)		
1.0	0.01	0.0	0.9		x		x		x		x			
		-0.2	1.0				x							
	0.05	0.0	0.0				x		x					
	0.1	0.0	0.0		x		x		x					
	0.2	0.0	0.0					x						
		-0.5	0.0	x										
	0.3	0.0	0.0				x		x					
			0.9		x				x					
2.0	0.01	0.0	0.0						x					

[a]Two sets of answers are given for each case. The first set represents the Sutherland solution for the indicated value of s, while the second set represents the solution using a power-law viscosity—temperature relation with $\omega = \omega_r$ as defined by Eq. (53b).

[b]Solution independent of s.

Key 6

Outer Solutions for $\beta \to \infty$

$I_1 \equiv 0$

ω	f_w	t_s	σ	$\left(\frac{u_e}{u_\infty}\right)^2$	Pr	t_w: 0	.15	.4	.5	.6	1.0
0.5	0.0	0.1000	0.9	1.0	0.7			x		x	
			1.0	0.0	0.7			x		x	
				0.5	0.7			x		x	
		0.3333	0.9	1.0	0.7		x	x		x	
			1.0	0.0	0.7		x	x		x	
				0.5	0.7		x	x		x	
		1.0	0.9	1.0	0.7		x	x		x	
			1.0	0.0	0.7		x	x		x	
				0.5	0.7		x	x		x	
	-0.2	1.0	0.9	1.0	0.7		x	x		x	
			1.0	0.0	0.7		x	x		x	
				0.5	0.7		x	x		x	
	-0.4	1.0	0.9	1.0	0.7		x	x		x	
			1.0	0.0	0.7		x	x		x	
				0.5	0.7		x	x		x	
0.7	0.0	0.1000	0.9	1.0	0.7		x	x		x	
			1.0	0.0	0.7		x	x		x	
				0.5	0.7			x		x	
		0.3333	0.9	1.0	0.7		x	x		x	
			1.0	0.0	0.7		x	x		x	
				0.5	0.7		x	x		x	
		1.0	0.9	1.0	0.7		x	x		x	
			1.0	0.0	0.7		x	x		x	
				0.5	0.7		x	x		x	
	-0.2	1.0	0.9	1.0	0.7		x	x		x	
			1.0	0.0	0.7		x	x		x	
				0.5	0.7		x	x		x	
	-0.4	1.0	0.9	1.0	0.7		x	x		x	
			1.0	0.0	0.7		x	x		x	
				0.5	0.7		x	x		x	
1.0	0.0	0.1000	1.0	1.0	1.0	x			x		x
		0.1538	1.0	1.0	1.0	x			x		x
		0.3333	1.0	1.0	1.0	x			x		x
		0.6250	1.0	1.0	1.0	x			x		x
		1.0	1.0	1.0	1.0	x			x		x

IV. NUMERICAL INTEGRATION PROCEDURE

The program for the numerical integration of the system of equations (Eqs. (40) to (43)) was written at RAND in FORTRAN for the IBM 7044. The system was treated as a two-point boundary-value problem, and the Runge-Kutta method was employed for the numerical integration. The sequence of operations is shown schematically in Fig. 2. Inputs to the program are values for the eight parameters and initial guesses for the four wall derivatives $f''(0)$, $\theta'(0)$, $g'(0)$, and $z'(0)$. The solutions are then separated into two categories: ordinary boundary-layer solutions and adiabatic wall solutions. The latter requires a subprogram that searches for and obtains solutions in which the adiabatic wall condition is met. In either category, the Runge-Kutta method is used to integrate the equation to a given value of η_{max}. On the basis of experience this value was selected to yield an acceptable asymptotic solution within the limits of the variable values at η_{max}. The integration stepsize was varied to insure the proper accuracy, and the values of the variables at η_{max} were held to within 10^{-5}.

When the integration procedure is completed, the final values at η_{max} are compared with the required values and the Newton-Raphson scheme is used to recalculate the initial condition. The procedure is as follows:

Let

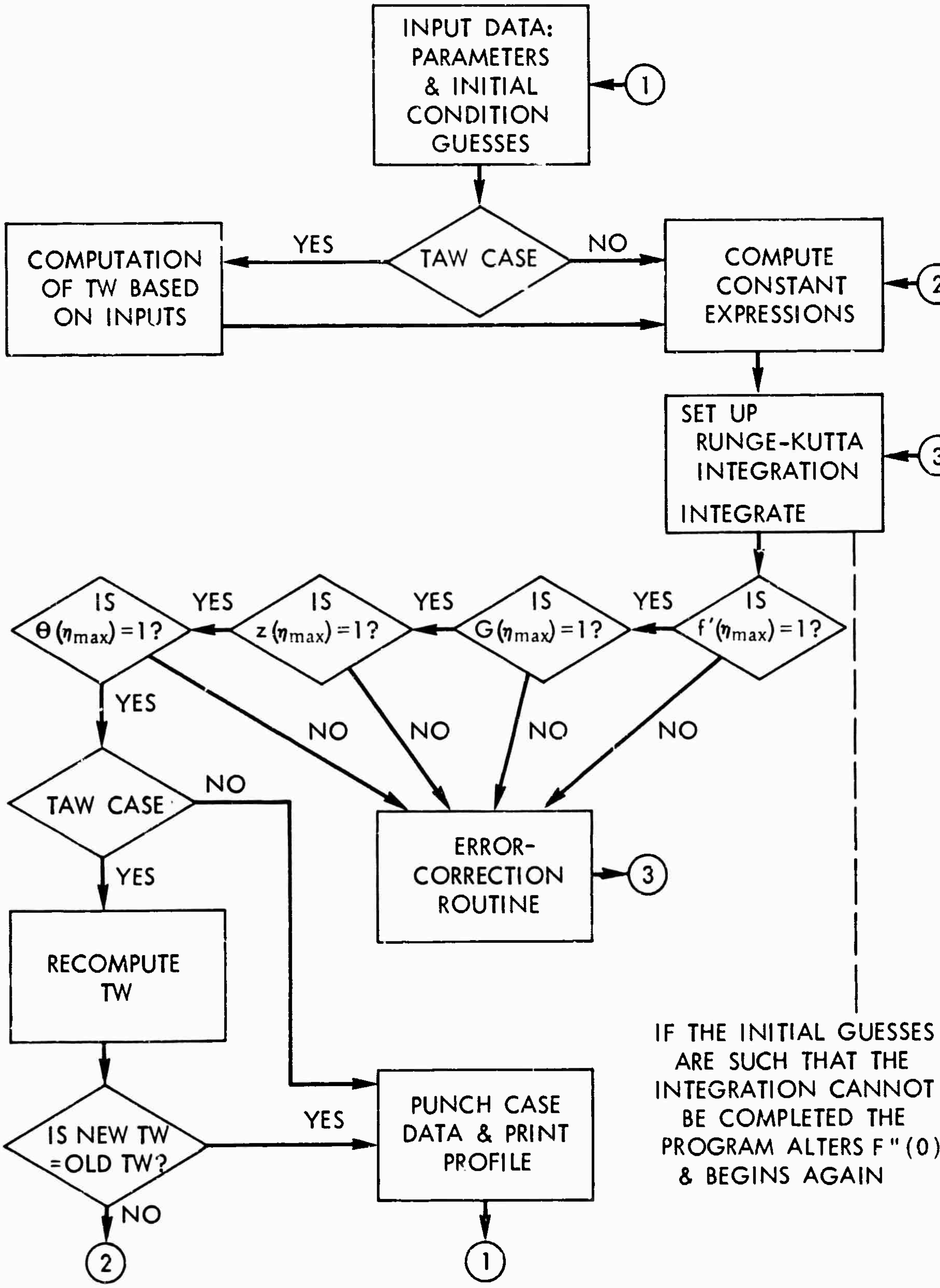

Fig. 2 -- Program flow chart.

$$\mathcal{E} = \mathcal{E}[f''(0),\ g'(0),\ z'(0),\ \theta'(0)] = f'(\eta_{max}) - 1 \tag{82}$$

$$\mathfrak{J} = \mathfrak{J}[f''(0),\ g'(0),\ z'(0),\ \theta'(0)] = g(\eta_{max}) - 1 \tag{83}$$

$$\mathfrak{D} = \mathfrak{D}[f''(0),\ g'(0),\ z'(0),\ \theta'(0)] = \theta(\eta_{max}) - 1 \tag{84}$$

$$\mathfrak{H} = \mathfrak{H}[f''(0),\ g'(0),\ z'(0),\ \theta'(0)] = z(\eta_{max}) - 1 \tag{85}$$

and

$$\mathfrak{e} = \begin{bmatrix} \mathcal{E} \\ \mathfrak{J} \\ \mathfrak{D} \\ \mathfrak{H} \end{bmatrix} ; \quad B = \begin{bmatrix} f'(\eta_{max}) \\ g\ (\eta_{max}) \\ \theta\ (\eta_{max}) \\ z\ (\eta_{max}) \end{bmatrix} ; \quad \text{and } S = \begin{bmatrix} f''(0) \\ g'(0) \\ \theta'(0) \\ z'(0) \end{bmatrix} \tag{86}$$

so that $\mathfrak{e} = \mathfrak{e}(S) = B(S) - 1 = \Delta B$. It is required that $\mathfrak{e} = \mathfrak{e}(\mathcal{E}, \mathfrak{J}, \mathfrak{D}, \mathfrak{H}) \to 0$. Now $d\mathfrak{e} = dB = M \cdot dS$, where

$$M = \begin{bmatrix} \dfrac{\partial f'_b}{\partial f''(0)} & \dfrac{\partial f'_b}{\partial g'(0)} & \dfrac{\partial f'_b}{\partial \theta'(0)} & \dfrac{\partial f'_b}{dz'(0)} \\ \dfrac{\partial g_b}{\partial f''(0)} & \dfrac{\partial g_b}{\partial g'(0)} & \dfrac{\partial g_b}{\partial \theta'(0)} & \dfrac{\partial g_b}{dz'(0)} \\ \dfrac{\partial \theta_b}{\partial f''(0)} & \dfrac{\partial \theta_b}{\partial g'(0)} & \dfrac{\partial \theta_b}{\partial \theta'(0)} & \dfrac{\partial \theta_b}{\partial z'(0)} \\ \dfrac{\partial z_b}{\partial f''(0)} & \dfrac{\partial z_b}{\partial g'(0)} & \dfrac{\partial z_b}{\partial \theta'(0)} & \dfrac{\partial z_b}{\partial z'(0)} \end{bmatrix} \tag{87}$$

The integration is performed using the initial guesses for S. This gives B and $\mathfrak{e}_o(S)$. The initial guesses are then perturbed as follows:

$$\mathcal{E}[f''(0) + \Delta f''(0),\ g'(0),\ \theta'(0),\ z'(0)] \tag{88}$$

$$\mathfrak{J}[f''(0),\ g'(0) + \Delta g'(0),\ \theta'(0),\ z'(0)] \tag{89}$$

$$\mathfrak{Z}[f''(0),\ g'(0),\ \theta'(0) + \Delta\theta'(0),\ z'(0)] \tag{90}$$

$$\mathfrak{H}[f''(0),\ g'(0),\ \theta'(0),\ z'(0) + \Delta z'(0)] \tag{91}$$

These integrations give the columns of M. The new guesses for the initial condition are corrected by calculating:

$$\Delta S = -M^{-1} \mathcal{E}_o \tag{92}$$

It was established that convergence was very difficult for cases involving high β ($\beta \gtrsim 2$). In these cases, the 10^{-5} limit accuracy was relaxed, and this is indicated in each table as needed.

V. APPLICATIONS OF THE CONCEPT OF LOCAL SIMILARITY

GENERAL DISCUSSION

The numerical results tabulated in this Memorandum are rigorously applicable only when the numerous similarity requirements listed in Section II are satisfied. In considering the diverse applications of compressible laminar boundary-layer theory, it is a rare occurrence indeed when all of these conditions are met. A most important question then arises: What approach should be used in predicting the behavior of the nonsimilar laminar boundary layer?

The copious literature relating to this question offers four basic types of approach. The first is to abandon the similar solutions entirely and adopt approximate techniques such as integral and series solutions containing free parameters. The most successful of the integral approaches appears to be that developed by Tani.[(35)] The transcendental approximation proposed by Hanson and Richardson[(36)] and the "improved approximation" technique of Yang[(37)] also appear very promising. Related to the integral methods is the powerful "strip method" proposed by Pallone,[(38)] which follows closely the inviscid flow integral method of Belotserkovski.[(39)]

A second type of nonsimilar calculation employs a strictly numerical approach. The complete nonsimilar boundary-layer equations are used and a new set of calculations is performed for each particular problem. Examples of this approach may be found in the works of Smith and Clutter[(40)] and Flügge-Lotz and Baxter.[(41,42)] Although numerically satisfying, these calculations are extremely expensive and intractable to generalization.

The third type of approach, suggested by Lees,[43] has been most successful in capturing the spirit of the use of similar solutions in situations where exact similarity does not exist. He observed that under certain circumstances, notably when there is a highly-cooled body in hypersonic flow, the local pressure-gradient parameter, β, had a negligible effect on the heat transfer to the surface. Many elaborations of this approach have been proposed to improve Lees' simple result to provide more accurate numerical estimates of heat transfer, skin friction, and boundary-layer thickness. Moore[22] gives a lucid summary of one group of these results. Additional ideas for modifying local similarity not discussed by Moore may be found in the papers of Smith,[44] Kemp, Rose, and Detra,[45] and Beckwith and Cohen.[6]

This third approach is saddled with one significant difficulty: it is necessary to make one or more implicit <u>ad hoc</u> approximations regarding the contribution of the nonsimilar terms in the complete boundary-layer equations. In each of the papers cited in the previous paragraph, the question is <u>not</u> "Are similar solutions applicable?" but rather "Which similar solution should be used?" A number of methods have been proposed for choosing the similar solutions most appropriate to the local inviscid flow conditions, local wall temperature, and boundary-layer history. In the context of the present discussion, this question necessitates the judicious choice of values for the eight similarity parameters. These values are usually determined (e.g., Ref. 6) by satisfying one or more integral conservation equations exactly using assumed profiles obtained from similar solutions. Such procedures are closely related to the integral method of Thwaites[46] and to the more recent use by Lees and Reeves[47] of a family of similarity profiles generated by Stewartson.[33]

The fourth, and in many ways most satisfying, type of approach to nonsimilar boundary-layer calculations may be traced to the work of Meksyn.[19] His underlying premise is very powerful: if the boundary layer is at all times very nearly described by a similar solution, then the direct effects of the nonsimilar terms may be calculated by asymptotically expanding the full boundary-layer equations in terms of small parameters which measure the departure of the solutions from similarity. In this way, the accuracy of local similarity methods is explicitly determined by using the full nonsimilar equations. We shall demonstrate shortly that the linearized equations governing the departure from similarity depend only on the local similarity parameters and, consequently, need be computed only once.

ASYMPTOTIC EXPANSION OF THE BOUNDARY-LAYER EQUATIONS

For purposes of illustration, we shall limit our consideration to the "incompressible" momentum equation

$$f_{\eta\eta\eta} + ff_{\eta\eta} + \beta(\xi)[1 - f_\eta^2] = 2\xi[f_\eta f_{\xi\eta} - f_\xi f_{\eta\eta}] \tag{93}$$

where ξ, η, and $\beta(\xi)$ are as defined in Section II and the subscripts η and ξ denote derivatives. Equation (93) may be obtained from the general equations by assuming $Pr = \omega = t_s = t_w = 1$. We also assume that $f_w = 0$, making the three boundary conditions for Eq. (93)

$$\left.\begin{aligned} f(\xi,0) &= 0 \\ f_\eta(\xi,0) &= 0 \\ f_\eta(\xi,\infty) &= 1 \end{aligned}\right\} \tag{94}$$

Merk[49] was the first to expand the complete nonsimilar momentum equation in terms of a small parameter. Subsequently Bush[50] pointed out that Merk's derivation neglected important terms in the correction equation. The remainder of this section will be concerned with Bush's equations and their approximate solution.

In the spirit of Meksyn's approximation, we look for solutions to Eq. (93) when the right-hand side is small. The key to an appropriate expansion is the inversion introduced by Merk. We change variables from $[\xi,\eta;\ \beta(\xi)]$ to $[\beta,\eta;\ \xi(\beta)]$ so that the streamwise momentum equation becomes

$$f_{\eta\eta\eta} + ff_{\eta\eta} + \beta[1 - f_\eta^2] = \epsilon(\beta)[f_\eta f_{\beta\eta} - f_\beta f_{\eta\eta}] \tag{95}$$

$$f(\beta,0) = f_\eta(\beta,0) = 0\ ; \qquad f_\eta(\beta,\infty) = 1 \tag{96}$$

where

$$\epsilon(\beta) = 2\xi\beta'(\xi) = 2\xi(\beta)/\xi'(\beta) \tag{97}$$

If ϵ is zero, the momentum equation reduces to the Falkner-Skan similarity equation with β as a single parameter. For small ϵ we may perform an asymptotic expansion of $f(\beta,\eta)$ of the form

$$f(\beta,\eta) = f_o(\beta,\eta) + \epsilon(\beta)f_1(\beta,\eta) + \ldots. \tag{98}$$

Then the derivatives f_η and f_β are

$$f_\eta = (f_o)_\eta + \epsilon(\beta)(f_1)_\eta + \ldots; \tag{99}$$

$$f_\beta = (f_o)_\beta + \epsilon(\beta)(f_1)_\beta + \ldots$$

$$+ \epsilon'(\beta) f_1 + \ldots \qquad (100)$$

As Bush pointed out, the term $\epsilon'(\beta)$ is, in general, of order unity and may be expressed as

$$\epsilon'(\beta) = \frac{1}{\beta'(\xi)} \frac{d}{d\xi}[2\xi\beta'(\xi)] \equiv 2[1 + E(\beta)] \qquad (101)$$

where

$$E(\beta) = \xi\beta''(\xi)/\beta'(\xi) = -\xi(\beta)\xi''(\beta)/[\xi'(\beta)]^2 \qquad (102)$$

Substitution of the asymptotic sequence [Eq. (98)] into the momentum equation and boundary conditions produces a hierarchy of equations, the first two of which are (primes on f_o and f_1 denote differentiation with respect to η):

Order Unity

$$\left.\begin{aligned} f_o''' + f_o f_o'' + \beta[1 - (f_o')^2] &= 0 \\ f_o(\beta,0) = f_o'(\beta,0) = 0 \; ; \quad f_o'(\beta,\infty) &= 1 \end{aligned}\right\} \qquad (103)$$

Order ϵ

$$\left.\begin{aligned} f_1''' + f_o f_1'' - A_1 f_o' f_1' + A_2 f_o'' f_1 &= \Phi(\beta,\eta) \\ f_1(\beta,0) = f_1'(\beta,0) = f_1'(\beta,\infty) &= 0 \end{aligned}\right\} \qquad (104)$$

In Eqs. (104) the terms (A_1, A_2, Φ) are

$$\left.\begin{aligned} A_1 &= 2 + 2\beta + 2E \\ A_2 &= 3 + 2E \\ \Phi(\beta,\eta) &= f_o'(f_o')_\beta - (f_o)_\beta f_o'' \end{aligned}\right\} \qquad (105)$$

Note that two independent parameters (β,E) appear in the first-order equation.

Equations (103) represent the similar solutions of Falkner and Skan.[51] Equations (104) represent the first correction f_1 to the velocity profile which arises from nonsimilar terms. For example, the skin-friction derivative $f''(\beta,0)$ is expressed as

$$f''(\beta,0) = f_o''(\beta,0) + \epsilon f_1''(\beta,0) + \ldots \qquad (106)$$

where $f_o''(\beta,0)$ is the local similarity solution corresponding to the local value of β, and $f_1''(\beta,0)$ is the correction obtained as a solution of Eqs. (104). It is apparent that the correction will be of order ϵ as long as $f_1''(\beta,0)$ is of the same order as $f_o''(\beta,0)$ and ϵ is small.

DETERMINATION OF $f_1(\beta,\eta)$ BY SUCCESSIVE APPROXIMATIONS

We proceed to a consideration of the first-order equations for $f_1(\beta,\eta)$, Eqs. (104). The differential equation is linear with homogeneous boundary conditions and may be solved numerically. The primary difficulty arises in computing the inhomogeneous term $\Phi(\beta,\eta)$ which contains derivatives of f_o with respect to both η and β. This is a difficult term to calculate numerically because a number of similarity

solutions in the neighborhood of β must be known with high precision. The exact numerical calculation of f_1 appears possible but has not been attempted.

The technique adopted here is to substitute a transcendental approximation for $f_o(\beta,\eta)$ which allows the coefficients of the differential equation and the forcing function $\Phi(\beta,\eta)$ to be expressed in terms of known functions. Following an earlier paper by Bush,[52] we represent f_o by the relation

$$f_o'(\beta,\eta) = \mathrm{erf}(t) \tag{107}$$

where

$$t = a\eta \tag{108}$$

$$a(\beta) = \frac{1}{2}\left[1 + \beta\left(1 + \frac{2}{\pi}\right)\right]^{1/2} \tag{109}$$

This relation is found to be in excellent agreement with exact solutions for f_o and appears quite adequate for our present purposes. It is convenient to transform coordinates from (β,η) to (a,t) and define a new dependent variable $g(a,t)$ according to the relations

$$\frac{\partial}{\partial\eta} = a\frac{\partial}{\partial t} \quad ; \quad \frac{\partial}{\partial\beta} = a'(\beta)\left(\frac{t}{a}\frac{\partial}{\partial t} + \frac{\partial}{\partial a}\right) \tag{110}$$

$$f_1(\beta,\eta) = \chi(a)g(a,t) \tag{111}$$

$$\chi(a) = \frac{2}{a^2\sqrt{\pi}}\left(\frac{da}{d\beta}\right) = \frac{0.23084}{a^3} \tag{112}$$

Substitution of these transformations into Eq. (104) results in a linear differential equation for g(a,t) which contains derivatives of g with respect to t up to third order. The boundary conditions for this equation are (primes denote differentiation with respect to t):

$$g(a,0) = g'(a,0) = g'(a,\infty) = 0 \tag{113}$$

The differential equation for g(a,t) is now integrated formally three times with respect to t, using the boundary conditions given by Eq. (113). The resulting integral equation is

$$g(a,t) = \frac{t^2}{2} g''(a,0) - \int_0^t J_1(t) g(a,t)\, dt + A_3 \int_0^t\!\!\int J_0(t) g(a,t)\, dt$$

$$-A_4 \int_0^t\!\!\int\!\!\int J_{-1}(t) g(a,t)\, dt + \psi_1(t) \tag{114}$$

The coefficients $A_3(a)$ and $A_4(a)$ are given by

$$A_3(a) = 4 + 2E + 2\beta \quad ; \quad A_4(a) = 6 + 4E + 2\beta \tag{115}$$

and the terms $J_n(t)$ are the n^{th} integrals of the error function:

$$\underline{n = -1,0} \qquad J_{-1}(x) = \frac{2}{\sqrt{\pi}} e^{-x^2} \quad ; \quad J_0(x) = \mathrm{erf}(x)$$

$$\underline{n = 1,2\ldots} \qquad J_{n+1}(x) = \frac{1}{1+n}\left[xJ_n(x) + \frac{1}{2} J_{n-1}(x) - \frac{x^n}{n!\sqrt{\pi}} \right] \tag{116}$$

The term $\psi_1(t)$ is defined by

$$\psi_1(t) = \frac{1}{2\sqrt{2}} J_2(\sqrt{2t}) - \frac{1}{2} J_2(t) - \frac{\sqrt{\pi}}{8} J_o(t)J_1(t) + \frac{1}{4} \psi_2(t) \qquad (117)$$

where

$$\psi_2(t) = \frac{1}{\sqrt{2}} J_o(\sqrt{2t}) - \frac{1}{2} J_o(t) \left[1 + \frac{\sqrt{\pi}}{2} J_{-1}(t) \right] \qquad (118)$$

A method of successive approximations is now applied to Eq. (114), substituting trial functions $\tilde{g}(a,t)$ for $g(a,t)$ in the three integrals and continuing until a trial function $\tilde{g}$ is found that agrees satisfactorily with the function g computed from the integral equation. This scheme differs from Picard's method in that the sequence of trial functions $\tilde{g}$ used in the integrals are suitably-chosen integrable functions of t rather than the functions $g(a,t)$ obtained in the previous iteration step. Picard's method converges absolutely, but in practice it usually cannot be continued analytically beyond one or two iterations. In the present scheme, convergence depends on the choice of trial functions $\tilde{g}$ but the integrals may be evaluated in closed form.

Evaluation of the Velocity Profile

In this analysis based on the simplified momentum equation [Eq. (93)] we are most interested in the correction $f_1''(\beta,0)$ to the velocity gradient at the wall. It is therefore more convenient to work with the first derivative of Eq. (114), which is (after some rearrangement)

$$g'(a,t) = tg''(a,0) - A_5J_1(t)g(a,t) + A_6 \int_o^t J_1(t)g'(a,t)\,dt$$

$$+ A_4 \int_o^t\!\!\int J_o(t)g'(a,t)\,dt + \psi_3(t) \qquad (119)$$

where

$$A_5 = 3 + 2E \;; \qquad A_6 = 2 + 2E \tag{120}$$

$$\psi_3(t) = \frac{1}{2} J_1(\sqrt{2}t) - \frac{1}{2} J_1(t) - \frac{\sqrt{\pi}}{8} J_o^2(t) \tag{121}$$

In following the technique of successive approximations, the terms g and g' on the right-hand side of Eq. (119) must be replaced by the trial function $\tilde{g}$ and its derivative $\tilde{g}'$. The behavior of g and g' may be inferred from the boundary conditions $g(a,0) = g'(a,0) = g'(a,\infty) = 0$ and is sketched in Fig. 3.

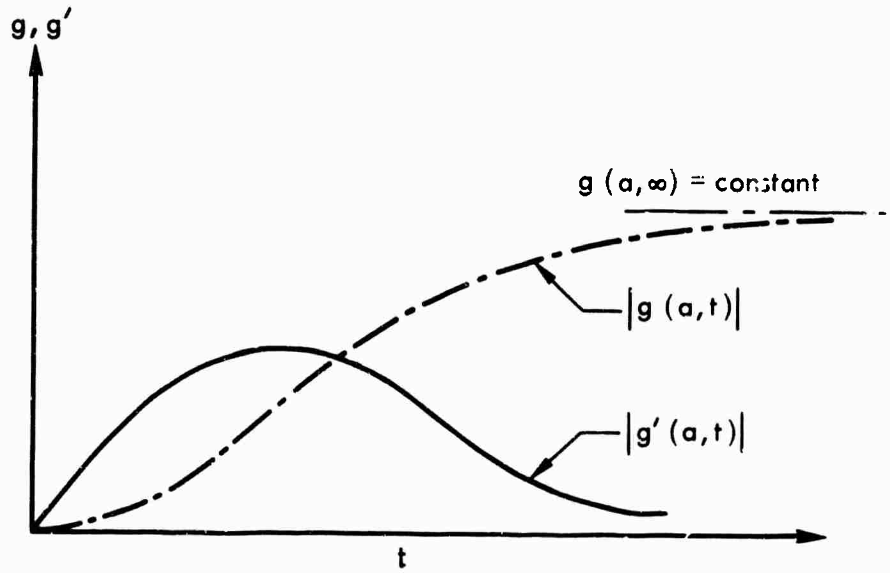

Fig. 3 -- Behavior of g and g'.

The behavior of $\psi_3(t)$ and the two integrals appearing in Eq. (119) may be inferred from the behavior of g' and the quantities $J_n(t)$.

Suppose we decompose the trial function $\tilde{g}(a,t)$ into two parts, so that

$$\tilde{g}(a,t) = C(a)\kappa(t) \tag{122}$$

We then define $\kappa(t)$ to be unity as $t \to \infty$, making $\tilde{g}(a,\infty) = C(a) \neq 0$. The boundary conditions $g(a,0) = 0$ and $g'(a,0) = 0$ are automatically satisfied for all suitable trial functions $\tilde{g}(a,t)$; the boundary condition $g'(a,\infty) = 0$ serves to evaluate both $C(a)$ and $g''(a,0)$. The terms appearing on the right-hand side of Eq. (119) have the following asymptotic forms:

$$\lim_{t\to\infty} [J_1(t)\tilde{g}(a,t)] = C(a)\left(t - \frac{1}{\sqrt{\pi}}\right) \tag{123}$$

$$\lim_{t\to\infty} \left[\int_0^t J_1(t)\tilde{g}'(a,t)\,dt\right] = C(a)\gamma_1 \tag{124}$$

$$\lim_{t\to\infty} \left[\int_0^t\int J_0(t)\tilde{g}'(a,t)\,dt\right] = C(a)\,[\gamma_2 t - \gamma_3] \tag{125}$$

$$\lim_{t\to\infty} [\psi_3(t)] = \frac{1}{2}(\sqrt{2} - 1)t - \frac{\sqrt{\pi}}{8} \tag{126}$$

The terms $(\gamma_1, \gamma_2, \gamma_3)$ are numerical constants which are determined by the choice of the trial function $\kappa(t)$.

Substituting Eqs. (123) to (126) into Eq. (119) and applying the boundary condition $g'(a,\infty) = 0$, we obtain the following formulas for $C(a)$ and $g''(0)$:

$$C(a) = \frac{\sqrt{\pi}}{8}\left[\frac{1}{\sqrt{\pi}} + A_6\left(\frac{1}{\sqrt{\pi}} + \gamma_1\right) - A_4\gamma_3\right]^{-1} \tag{127}$$

$$g''(a,0) = C(a)[(1 + A_5) - A_4\gamma_2] - \frac{1}{2}(\sqrt{2} - 1) \tag{128}$$

We have chosen several functions $\kappa(t)$ and used Eqs. (127) and (128) to determine C(a) and $g''(a,0)$. The integrals appearing in Eq. (119) have been evaluated in closed form and the profiles $g'(a,t)$ have been determined for each trial function.

Products and sums of the functions $J_n(t)$ are useful in generating successively more accurate trial functions $\kappa(t)$ because the integrals are easily evaluated. Typical examples of $\kappa(t)$ are:

$$\kappa(t) = \begin{cases} J_o(t) \\ J_o^2(t) \\ \left[1 - \frac{\sqrt{\pi}}{2} J_{-1}(t)\right] \\ \frac{2}{(\sqrt{2} - 1)} \psi_2(t) \end{cases} \tag{129}$$

and there are others.

Weighted sums of the functions listed above were also used to effect a close fit to $g'(a,t)$. The computed profiles $g'(a,t)$ differed in magnitude with the different functions $\kappa(t)$, but both the qualitative behavior and the quantity $g''(a,0)$ were relatively insensitive to the form of $\kappa(t)$.

For $\beta = E = 0$, $g'(a,t)$ has a maximum near $t = 0.84$ and decreases to about 12 percent of its maximum value at $t = 2.0$. The derivative $g''(a,0)$ is approximately -0.035. For $\beta = 0$, the term $a(\beta)$ computed from Eq. (109) is 1/2, and

$\underline{\beta = E = 0}$

$$f_1''(0,0) = a^2\chi(a)g''(a,0)$$

$$\cong -.025$$

The Blasius solution to Eqs. (103) for $\beta = 0$ is $f_o''(0,0) = 0.46960$, and the final estimate of the correction to the skin friction coefficient for $\beta = E = 0$ is

$$f''(0,0) \cong f_0''(0,0)[1 - 0.053\ \epsilon + \dots] \tag{130}$$

The physical interpretation of this result agrees qualitatively with the sign of the correction. If $\epsilon > 0$, β is increasing and the local similarity value, $f_o''(\beta,0)$, would neglect "relaxation" effects and overestimate the shear at the wall. The correction reduces the skin-friction value by an amount proportional to ϵ. The small magnitude (.053) of the correction term is somewhat surprising, but this value is probably accurate to within about ± 20 percent. Bush estimated the correction term to be 0.204 but his result was obtained by asymptotically expanding an approximate solution rather than obtaining an approximate solution to the exact first-order equation.

VI. SOLUTIONS FOR LARGE VALUES OF THE PRESSURE-GRADIENT PARAMETER β

In using the concept of local similarity in highly accelerated flows, it is useful to have solutions of the laminar boundary-layer equations for large values of β. The mathematical difficulties encountered in the limit $\beta \gg 1$ may be illustrated by rewriting the streamwise momentum equation [Eq. (40)] in the form

$$\frac{1}{\beta}\left[(\lambda f'')' + ff''\right] - \left\{f'^2 - \frac{1}{t_s}\left[(1 - t_w)\theta - (1 - t_s)g^2 + t_w\right]\right\} = 0 \quad (131)$$

Here λ is redefined as the density-viscosity ratio at the wall $(\rho\mu/\rho_w\mu_w)$.

In the limit $\beta \to \infty$, Eq. (131) reduces from third to first order:

$$\lim_{\beta\to\infty} f' = \left\{\frac{1}{t_s}\left[(1 - t_w)\theta - (1 - t_s)g^2 + t_w\right]\right\}^{1/2} \quad (132)$$

The transverse-momentum equation and the energy equation remain of second order. As originally pointed out by Coles[53] and as later elaborated upon by Beckwith and Cohen,[6] this leads to a singular perturbation problem in which the thickness of the velocity layer is of order $\beta^{-1/2}$ with respect to the total enthalpy layer and transverse velocity layers of order unity.* The method of solution is similar to that employed in deriving a uniformly valid approximation to the Navier-Stokes equations in the limit of large Reynolds number [see Kaplun,[55] Lagerstrom and Cole,[56] and the recent book by Van Dyke[57]].

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*Discussion of this problem also appears in an abbreviated version in Lagerstrom's article.[54]

THE OUTER LIMIT EQUATIONS

Since the mathematical justification of this singular perturbation solution (more popularly called an inner- and outer-expansion procedure) has been discussed in detail by C les,[53] our purposes will be served by a cursory development of the governing equations. The present analysis extends the work of Beckwith and Cohen[6] to include a power-law temperature-viscosity relation, a constant but nonunit Prandtl number, and the mass transfer at the surface.

Assume that appropriate outer representations of the dependent variables f, θ, g are of the following forms:

$$\left.\begin{aligned} f &= f_o + \frac{1}{\sqrt{\beta}} f_1 + \dots \quad ; \\ \theta &= \theta_o + \frac{1}{\sqrt{\beta}} \theta_1 + \dots \quad ; \\ g &= g_o + \frac{1}{\sqrt{\beta}} g_1 + \dots \end{aligned}\right\} \tag{133}$$

Substituting these representations into the momentum equations and the energy equations and dropping all terms of order $\beta^{-1/2}$ and smaller,* the following "outer" equations are obtained:

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*The outer limit equations are properly obtained by applying the limit $\beta \to \infty$ to the full equations expressed in outer variables, with η held fixed. This gives identical results to those cited here.

$$f_o' = \left\{ \frac{1}{t_s} \left[(1 - t_w)\theta_o - (1 - t_s)g_o^2 + t_w \right] \right\}^{1/2} \tag{134}$$

$$\left(\frac{\lambda}{Pr}\theta_o'\right)' + f_o\theta_o' + \left\{ \frac{2\lambda\sigma}{(1 - t_w)} \left(1 - \frac{1}{Pr}\right) \right.$$

$$\left. \times \left[\frac{f_o'^2}{2} \left(\frac{u_e}{u_\infty}\right)^2 \cos^2 \Lambda + \frac{g_o^2}{2} \sin^2 \Lambda \right]' \right\}' = 0 \tag{135}$$

$$(\lambda g_o')' + f_o g_o' = 0 \tag{136}$$

The appropriate boundary conditions are found (a) by requiring that the outer equations satisfy the exact outer boundary conditions, and (b) by exact matching of the inner representations with the outer. The results may be written

$$f_o(0) = f_w \quad ; \quad \theta_o(0) = g_o(0) = 0 \quad ; \quad \theta_o(\infty) = g_o(\infty) = 1 \tag{137}$$

Only one boundary condition on f_o may be satisfied by the outer equations, because the outer limit equation for f_o reduces to first order. The exact boundary condition $f' \to 1$ as $\eta \to \infty$ is automatically satisfied by Eq. (134). The no-slip condition, $f' \to 0$ as $\eta \to 0$, must be satisfied by an inner solution which is valid in a region of extent $\beta^{-1/2}$ with respect to the scale of the outer solution.

In this approximation, the density-viscosity ratio $\lambda = (\rho\mu/\rho_w\mu_w)$ in the outer layer becomes

$$\lambda = \left\{\frac{1}{t_w}\left(\frac{1 - \sigma_2}{t_s}\right)\left[(1 - t_w)\theta_o - (1 - t_s)g_o^2 + t_w\right]\right\}^{\omega-1} \tag{138}$$

where

$$\sigma_2 = \left(\frac{U_\infty^2}{2H_e}\right)\left[\left(\frac{u_e}{u_\infty}\right)^2 \cos^2\Lambda + \sin^2\Lambda\right]$$

As $\eta \to 0$, $\lambda \to [(1 - \sigma_2)/t_s]^{\omega-1}$ because $f_o'(0) = (t_w/t_s)^{1/2}$ and $f_o'(0)$ is nonzero in general.* The energy and transverse momentum equations are coupled through the implicit appearance of both θ_o and g_o in Eqs. (135) and (136). If Pr = 1, $\theta_o = g_o$, and the number of coupled ordinary differential equations is reduced from three to two. Setting both the viscosity-temperature exponent ω and the Prandtl number Pr equal to unity reproduces the equations of Beckwith and Cohen.[(6)] If $\sigma_2 = 0$ (i.e., the local Mach number is zero), $t_s = 1$, and the transverse momentum and energy equations are again uncoupled.

One interesting case was pointed out by Coles and we extend his result to generalized compressible flow. Let $t_w = t_s = 1$, $\Lambda = 0$ so that $\sigma_2 = (U_\infty^2/2H_e)(u_e/u_\infty)^2 \equiv \sigma_1$. Then $\lambda = (1 - \sigma_1)^{\omega-1}$ and $f_o' = 1$ for all η. In taking this limit with Pr $\neq$ 1, the product of $(f_o'^2)$ and $(1 - t_w)^{-1}$ approaches θ_o' so that the energy equation becomes

$$\theta_o'' + \theta_o'(\eta + f_w)\ \mathrm{Pr}\ (1 - \sigma_1)^{1-\omega}[1 - \sigma_1(1 - \mathrm{Pr})]^{-1} = 0 \tag{139}$$

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*It should be noted that the inner and outer expansion procedure breaks down in the limit $t_w \to 0$, because the outer solution for f_o satisfies the exact boundary conditions for the complete equations and the inner solution for f' is simply zero.

with the boundary conditions

$$\left.\begin{aligned} \eta \to 0 \;, &\quad \theta_o(0) = 0 \;; \\ \eta \to \infty \;, &\quad \theta_o(\infty) = 1 \;. \end{aligned}\right\} \tag{140}$$

If we define the new variable χ and the constants χ_o and W by

$$\left.\begin{aligned} \chi &= \frac{\eta}{\sqrt{W}} - \chi_o \;; \\ \chi_o &= -\frac{f_w}{\sqrt{W}} \;; \\ W &= [1 - \sigma_1(1 - Pr)][Pr(1 - \sigma_1)^{\omega-1}]^{-1} \end{aligned}\right\} \tag{141}$$

then Eqs. (139) and (140) are satisfied by the solution

$$\theta = [\mathrm{erf}(\chi/\sqrt{2}) + \mathrm{erf}(\chi_o/\sqrt{2})]/[1 + \mathrm{erf}(\chi_o/\sqrt{2})] \;; \tag{142}$$

For $\chi_c < 0$, we note the identity

$$\mathrm{erf}(-\chi) = -\mathrm{erf}(\chi) \tag{143}$$

Equations (134) to (136) with the boundary conditions of Eq. (137) represent a two-point boundary-value problem for the complete solution of the outer limit equations. The derivatives $\theta_o'(0)$ and $g_o'(0)$ along with the integrals I_2, $I_1(1)$, $I_1(2)$, and $I_1(3)$ evaluated using f_o, θ_o, and g_o in place of f, θ, and g are given in Table 6. The integral I_1 is identically equal to zero. Beckwith and Cohen[6] calculated several of these quantities for the case of $Pr = \omega = 1$, $f_w = 0$, and nonunit values of t_s.

THE INNER LIMIT EQUATIONS

Inasmuch as the order of the energy and transverse momentum equations is not reduced in taking the limit $\beta \to \infty$, the outer equations [Eqs. (134) to (136)] represent complete solutions for the total enthalpy and transverse velocity profiles for large β. The terms f_o and $(f_o'^2)'$, which appear in the outer equations, differ from the exact solutions f and $(f'^2)'$ only in a region which is $\beta^{-1/2}$ smaller in extent than the region of applicability of the outer equations. Therefore, the outer equations asymptotically represent the complete solutions for θ and g as $\beta \to \infty$, and in this limit, the inner solutions for θ and g are identically zero.

The no-slip condition $f'(0) = 0$ is satisfied by the inner limit equation for f; the inner equations for θ and g, as noted previously, reduce to $\theta = g = 0$. To examine the inner streamwise-momentum equation, it is necessary to introduce the new independent variable $\tilde{\eta}$ and an inner representation for f:

$$\tilde{\eta} = \sqrt{\beta}\,\eta \; ; \qquad f = f_w + \frac{1}{\sqrt{\beta}} \tilde{f}_o(\tilde{\eta}) + \ldots \tag{144}$$

so that

$$\tilde{f}_o'(\tilde{\eta}) = f_o'(\eta) = \frac{u}{u_e} \tag{145}$$

After rearrangement, the inner equation for $\tilde{f}_o'$ becomes

$$(\lambda \tilde{f}_o'')' + \frac{1}{\beta} \tilde{f}_o \tilde{f}_o'' - (\tilde{f}_o')^2 + \frac{t_w}{t_s} = 0 \tag{146}$$

and dropping terms of order $\beta^{-1/2}$ and smaller, the result is

$$(\lambda \tilde{f}_o'')' - (\tilde{f}_o')^2 + \frac{t_w}{t_s} = 0 \tag{147}$$

where

$$\lambda = \left[1 - \frac{\sigma_1 \cos^2 \Lambda}{t_w} (\tilde{f}_o')^2\right]^{\omega-1} \tag{148}$$

The boundary conditions on $\tilde{f}_o'$ are found from the exact boundary conditions at the wall and the matching condition* that the inner and outer representations of f agree in the limit $\sqrt{\beta} \to \infty$ with $\tilde{\eta}$ large but fixed. The following boundary conditions for the inner equation result:

$$\tilde{f}_o'(0) = 0 \quad ; \qquad \tilde{f}_o'(\infty) = \sqrt{t_w/t_s} \tag{149}$$

Since Eqs. (147) and (148) involve only $\tilde{f}_o'$ and not $\tilde{f}_o$, they represent a well posed, second-order, two-point boundary-value problem. A more symmetrical form may be obtained by one final transformation:

$$s_o(t) = \sqrt{\frac{t_s}{t_w}}\ \tilde{f}_o'(\tilde{\eta}) \quad ; \qquad t = (t_w/t_s)^{1/4}\ \tilde{\eta} \tag{150}$$

Then the problem may be written

$$(\lambda s_o')' - s_o^2 + 1 = 0 \tag{151}$$

---

*The matching condition applied here is elaborated by Van Dyke: Inner representation of "outer representation" = Outer representation of "inner representation".

with the boundary conditions

$$s_o(0) = 0 \; ; \qquad s_o(\infty) = 1 \tag{152}$$

and the definitions

$$\lambda = [1 - a s_o^2]^{\omega - 1} \tag{153}$$

$$a = \sigma_1 \cos^2 \Lambda / t_s \tag{154}$$

Values of $s_o'(0)$ satisfying Eqs. (151) to (153) are given in Table 7 for several values of ω, σ, and Λ. For the special case of $\omega = 1$, Coles[(53)] pointed out that an analytic solution may be found in the form

$$s_o = 1 - 3 \operatorname{sech}^2 \left(t/\sqrt{2} + \tanh^{-1} \sqrt{2/3}\right) \tag{155}$$

Using either the analytic solution for $\omega = 1$ or the numerical solutions for $\omega \neq 1$, the surface skin friction derivative f_w'' is then found by reversing the previous transformations

$$\lim_{\beta \to \infty} f_w'' = \sqrt{\beta} \left(\frac{t_w}{t_s}\right)^{3/4} s_o'(0) \tag{156}$$

In general $s_o'(0)$ depends on the three parameters σ_2, ω, and t_s; for $\omega = 1$, the value of $s_o'(0)$ is 4/3.

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*Our thanks go to J. Aroesty of The RAND Corporation for suggesting a simple numerical quadrature technique for solving Eq. (151).

VII. DISCUSSION

The large number of the solutions in Tables 1 to 7 (p. 78 ff.) suggests many possibilities for numerical correlations and comparisons with approximate results. Although an exhaustive discussion of these possibilities goes beyond our present purposes, we present selected examples in Figs. 4 to 14 (p. 72 ff.) of the use of the present solutions in understanding the influence of the similarity parameters on heat, mass, and momentum transfer.

The local skin-friction and heat-transfer coefficients C_h and C_f are defined by Eqs. (70) and (73). Following Ref. 59, they may be placed in a more symmetric form by using the Reynolds number

$$Re_{e,x} = \rho_e u_e x/\mu_e \tag{157}$$

and the dimensionless streamwise coordinate

$$\begin{aligned} \tilde{\xi} &= \xi(\rho_w \mu_w u_e r_k^{2j} x)^{-1} \\ &= \left(\int_0^x \rho_w \mu_w u_e r_k^{2j}\, dx\right)\left(\rho_w \mu_w u_e r_k^{2j} x\right)^{-1} \end{aligned} \tag{158}$$

With these definitions, the quantities C_h and C_f become

$$C_h = \frac{\cos\Lambda}{\sqrt{2\tilde{\xi} Re_{e,x}}}\left(\frac{\rho_w \mu_w}{\rho_e \mu_e}\right)^{1/2}\left(\frac{\rho_e u_e}{\rho_\infty u_\infty}\right)\left[\frac{\theta_w'(1 - t_w)}{Pr(t_{aw} - t_w)}\right] \tag{159}$$

$$C_f = \frac{2\cos\Lambda}{\sqrt{2\tilde{\xi} Re_{e,x}}}\left(\frac{\rho_w \mu_w}{\rho_e \mu_e}\right)^{1/2}\left(\frac{\rho_e u_e}{\rho_\infty u_\infty}\right)\left[(f_w'')^2\left(\frac{u_e}{u_\infty}\right)^2 \cos^2\Lambda + (g_w')^2 \sin^2\Lambda\right]^{1/2} \tag{160}$$

In Eqs. (159) and (160), only the terms appearing in the square brackets depend upon the similarity parameters; the terms preceding the square brackets represent the external flow conditions and wall temperature. The Reynolds analogy function

$$j = 2C_h/C_f \qquad (161)$$

is simply the ratio of the square brackets appearing in C_h and C_f and consequently depends only upon the similarity parameters.

In Fig. 4, the expected increase in the skin-friction coefficient with increasing pressure-gradient parameter β is shown. The relative increase in $C_f/(C_f)_{\beta=0}$ is much greater for higher wall temperatures. This occurs because the boundary layer is thicker and the response to increasing β is proportionately higher.

Figure 5 shows similar behavior for the heat-transfer coefficient. We have used the wall-gradient parameter $\theta_w'[(1 - t_w)/(t_{aw} - t_w)]$ in forming the heat-transfer coefficient. As shown previously,[4] θ_w' varies greatly with β for $t_w > 0.6$ because t_{aw} varies with β if $\sigma \neq 0$.* The percentage increase here is less than for the skin friction but the trends with β and t_w are the same. Lees[43] argued that the high density in the boundary layer near the wall for low wall temperatures "insulates" the wall from pressure-gradient effects and the influence of

*It is particularly important to determine the proper adiabatic wall temperature in cases with mass injection because t_{aw} is greatly decreased by injection (see Table 3). The assumption Pr = 1 predicts $t_{aw} = 1$ under all conditions, which is seriously in error for large injection and high local Mach numbers.

β on C_h is greatly reduced. This behavior is shown with decreasing values of t_w.

The next two figures deal with the Reynolds analogy function $j = 2C_h/C_f$. Li and Gr [60] showed earlier that large deviations from unity could occur in a hypersonic boundary layer even for $t_w \ll 1$ and $\beta = 0$.

Figure 6 demonstrates that j decreases with increasing β, the decrease being faster with increasing wall temperature. This result can be easily explained by examining the limiting equations for $\beta \to \infty$. The heat-transfer coefficient C_h approaches an asymptote as $\beta \to \infty$, whereas for large β the skin-friction coefficient increases as $\beta^{1/2}$. The approach to this limiting behavior is more rapid with larger wall temperatures. Similar behavior is shown in Fig. 7 for values of the hypersonic parameter $\sigma = 0$, 0.5, and 1.0.

Figure 8 shows the well known boundary-layer property that heat transfer is decreased by mass injection at the surface. The heat-transfer reduction caused by a given value of f_w decreases with increasing β. The reason will be explained below in the discussion on limiting solutions for large β.

Finally, the sweep angle parameter, t_s, is varied in Fig. 9. When the sweep angle Λ is zero, there is skin friction only in the x-direction (see Eq. (160)). As the sweep angle increases, the spanwise term, $g'(0)\sin \Lambda$, contributes increasingly to the skin-friction coefficient. If the free-stream direction is sufficiently oblique, the x-component of C_f, $f''(0)\,(u_e/u_\infty)\cos \Lambda$, no longer exerts an appreciable influence. The quantity

$$\{1 + [g''(0) \sin \Lambda / f''(0) \cos \Lambda]^2\}^{-1/2}$$

plotted in Fig. 9 is a relative measure of the contribution of the two skin-friction components, and approaches a limiting value as $t_s \to 0$. Whalen[61] reported a similar result for the displacement thickness for $Pr = \omega = 1$.

It is very difficult to obtain exact numerical solutions of the laminar-boundary-layer equations for values of β greater than 2. The reason is simply that the singular behavior of $f'(\eta)$ near the wall as $\beta \to \infty$ becomes dominant even for moderate values of β. Conversely, the results obtained for $\beta = \infty$ should be good representations of the behavior of the boundary layer for large but finite values of β. One of the important results we wish to demonstrate is that, by combining the exact numerical results for $\beta \leq 5$ given in Tables 1 to 5 and the limiting solutions for $\beta \to \infty$ given in Tables 6 and 7, it is possible to estimate accurately the skin-friction and heat-transfer derivatives f''_w, g'_w, and θ'_w for all positive values of β.

The skin-friction results for $\beta = \infty$ are displayed in Fig. 10. The influences of the two parameters ω and a on the inner solutions for $s'_o(0)$ are seen to be relatively small. For $0.5 \leq \omega \leq 1.0$ and $0 \leq a \leq 1.0$, the value of $s'_o(0)$ may be found by interpolation to better than 0.25 percent.

The skin-friction parameter $f''_w \beta^{-1/2}(t_w/t_s)^{-3/4}$ approaches the limit of $s'_o(0)$ as $\beta \to \infty$. The difficulties that were observed previously in calculating exact solutions for $\beta > 2$ imply that this limit is approached very rapidly with increasing β. This supposition is borne out by Figs. 11 to 13, where the skin friction parameter is

shown as a function of $\beta^{-1/2}$. The limit parameter $\beta^{-1/2}$ is suggested by the ordering procedure used to obtain the inner and outer equations. Solid lines indicate exact numerical solutions and dashed lines are extrapolations.

Figure 11 illustrates the approach of the skin-friction parameter to its asymptotic limit $\tilde{f}_o'(0)$ for different wall temperatures. The limiting value is approached most rapidly for high wall temperatures. This result is to be expected from the behavior of the outer equations. Large values of t_w increase the magnitude of the velocity difference across the inner layer, whereas for $t_w \to 0$, the distinction between the inner and outer layers breaks down and no proper limit is obtained. It has also been found empirically that exact solutions are more difficult to obtain for large t_w.

The approach of the skin-friction parameter to its limiting value with increasing β is illustrated in Fig. 12 for several values of the sweep parameter t_s. From a numerical point of view, the accuracy of the present extrapolation procedure increases with increasing sweep.

It is of interest to examine the behavior of the inner and outer equations with mass injection at the wall. Applications of these results include mass-transfer cooling of rocket nozzles (Back and Witte)[57] and blunt hypervelocity vehicles. The outer equations determine the heat-transfer derivative θ_w', and they contain the boundary condition $f_o(\eta \to 0) = f_w$. Thus, surface mass transfer ($f_w < 0$) acts to reduce θ_w' and, consequently, surface-heat transfer even in the limit $\beta \to \infty$. On the contrary, the skin friction derivative f_w'' is found from the inner equation for $\tilde{f}_o'(0)$, Eq. (147), a the solution of this equation is independent of the value of f_w. In highly accelerated flows,

therefore, the effects of blowing on skin friction become negligible in the limit $\beta \to \infty$ with f_w fixed.

Figure 13 illustrates the behavior of the skin-friction parameter as a function of the injection parameter f_w. The limit is approached smoothly with decreasing values of $\beta^{-1/2}$ for all f_w, and accurate estimates of f''_w may be obtained for all β by comparing the exact solutions for $\beta \le 5$ and the inner limit solutions of Fig. 10.

The wall heat transfer is related by the modified Stewartson and Howarth-Dorodnitsyn transformations (Eqs. (10) and (11)) to θ'_w. Inasmuch as θ_o anf g_o represent the complete solutions for θ and g as $\beta \to \infty$, the values of $\theta'_o(0)$ and $g'_o(0)$ represent the asymptotic limits of θ'_w and g'_w as $\beta \to \infty$. It should be emphasized that θ'_w and g'_w become independent of β as $\beta \to \infty$, demonstrating that the heat transfer predicted by a local similarity analysis in highly accelerated flows approaches a limiting value.

Figure 14 shows the typical behavior of the heat-transfer derivative θ'_w with increasing values of β. The approach of θ'_w to its limiting value for $\beta \to \infty$, is seen to be smooth and (at least for the case of $\sigma_1 = 0$) monotonic. In an earlier paper,[4] we demonstrated that the proper parameter to use in comparing different heat-transfer calculations is $\theta'_w[(1 - t_w)/(t_{aw} - t_w)]$. For $\sigma_1 = 0$ and $t_s = 1$, the adiabatic wall temperature t_{aw} is unity for all Pr so that the heat transfer parameter reduces to θ'_w. Although we have not been able to prove it analytically, it appears that $t_{aw} = 1.0$ for all values of Pr, t_s, ω, and σ in the limit $\beta = \infty$. This is a very surprising result and should be examined further.

The boundary-layer displacement thickness δ^* is defined by the relation [Eq. (79)]

$$\delta^* \equiv \int_0^\infty \left(1 - \frac{\rho u}{\rho_e u_e}\right) dy = \frac{\sqrt{2\xi}}{\rho_e u_e r_k^j} \left(\frac{T_o}{T_e}\right) \left[I_1 - \left(\frac{T_e}{T_o}\right) I_2 \right]$$

where the integrals I_1 and I_2 are given in the list of symbols. For $\beta = \infty$, the integral I_1 is identically zero and the integral I_2 [Eq. (75)]

$$I_2 = \int_0^\infty f'(1 - f')\, d\eta$$

is recorded in Table 6. In the absence of sweep, the velocity profile is monotonic and $0 < f' < 1$, so that the integral I_2 is positive and the displacement thickness is negative. With sweep ($t_s < 1$), there is often an overshoot in the velocity profile, so that $f' > 1$ for some range of η, and I_2 becomes negative. With sweep, therefore, the displacement thickness may be either positive or negative, depending upon the particular parameters being considered. Numerical comparison between the calculated results for moderate β and the present results for $\beta = \infty$ suggests that δ^* monotonically decreases with increasing β. In the special case when $\beta = \infty$ and $t_w = t_s = 1$, $f' = 1$ and δ^* is of the order of $\beta^{-1/2}$ times the scale of the boundary displacement layer thickness for $\beta = 0$.

The numerical results given in Table 5 display the variation of boundary-layer properties for various Prandtl numbers near unity. These results may be used to estimate recovery factors for $Pr > 1$ by

combining the present results with the asymptotic solution of Narasimha and Vasantha[61] for Pr >> 1. Although these authors explicitly solved the problem of a flat plate ($\beta = 0$) with $\omega = 1$ and low Mach numbers, there is no reason why a similar analysis could not be conducted for $\beta > 0$ and general compressible flow. As Narasimha and Vasantha demonstrate, interpolations accurate to better than 3 percent in the recovery factor can be made between first-order asymptotic solutions for Pr >> 1 and exact numerical results for Pr of order unity.

In concluding this discussion, it is advisable to point out that the numerical values listed for the wall derivatives f_w'', θ_w', and g_w' allow reconstruction of the complete velocity and total enthalpy profiles by standard numerical integration techniques. Whereas the boundary-layer equations themselves, including the boundary conditions at $\eta = 0$ and $\eta \to \infty$, represent a two-point boundary-value problem, one may solve the equations as an ititial-value problem if the wall derivatives f_w'', θ_w', and g_w' are known. Furthermore, by using the wall derivatives given here, multi-term expansions of the boundary-layer equations may be constructed for small η. This property is useful in cases where additional "nonclassical" behavior occurs near the surface, for example, in MHD boundary layers where an electrostatic potential sheath exists for small η.

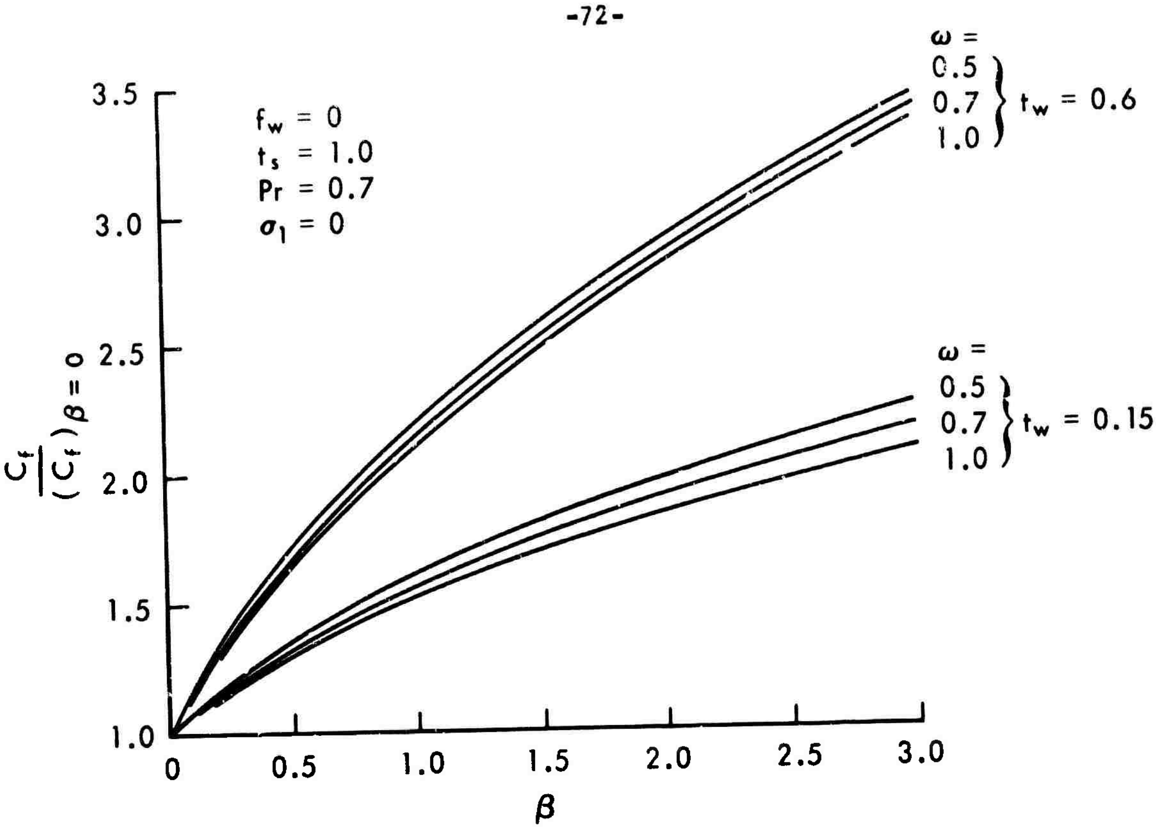

Fig.4—Variation of the skin friction coefficient with pressure gradient

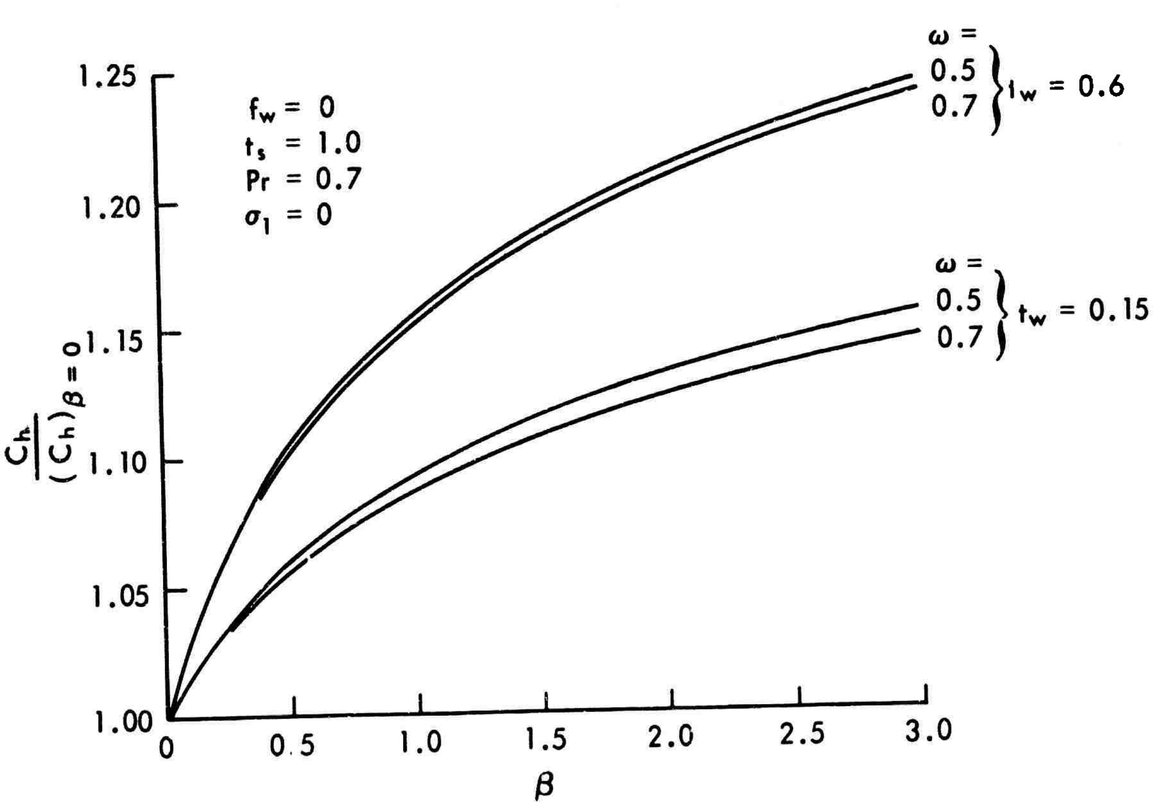

Fig.5—Variation of the heat transfer coefficient with pressure gradient

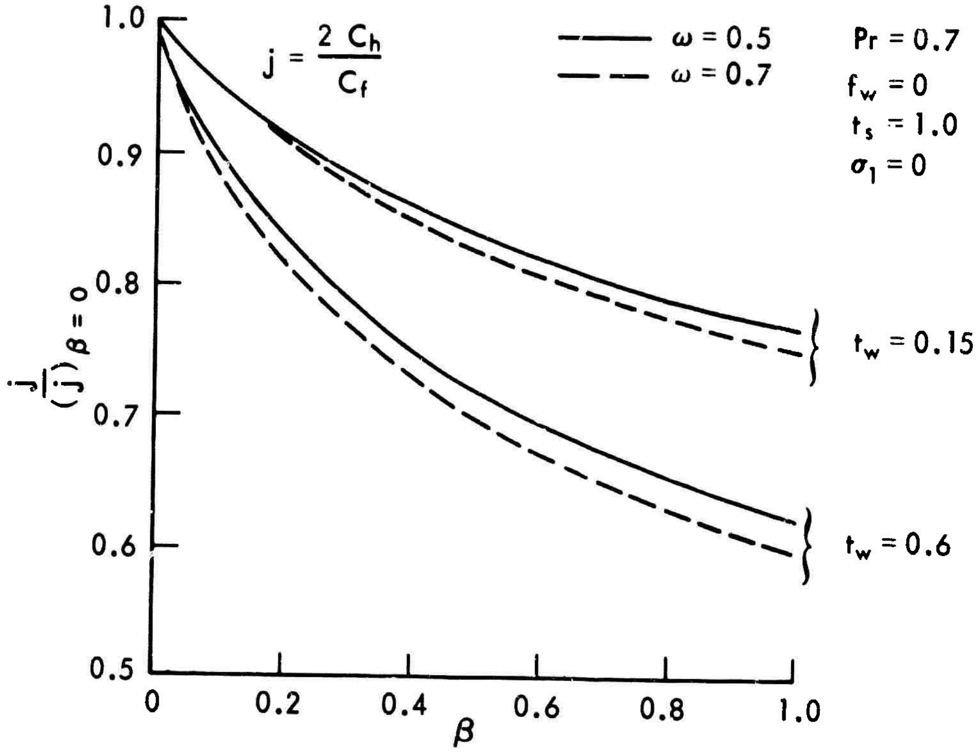

Fig.6—Variation of the Reynolds analogy function with pressure gradient

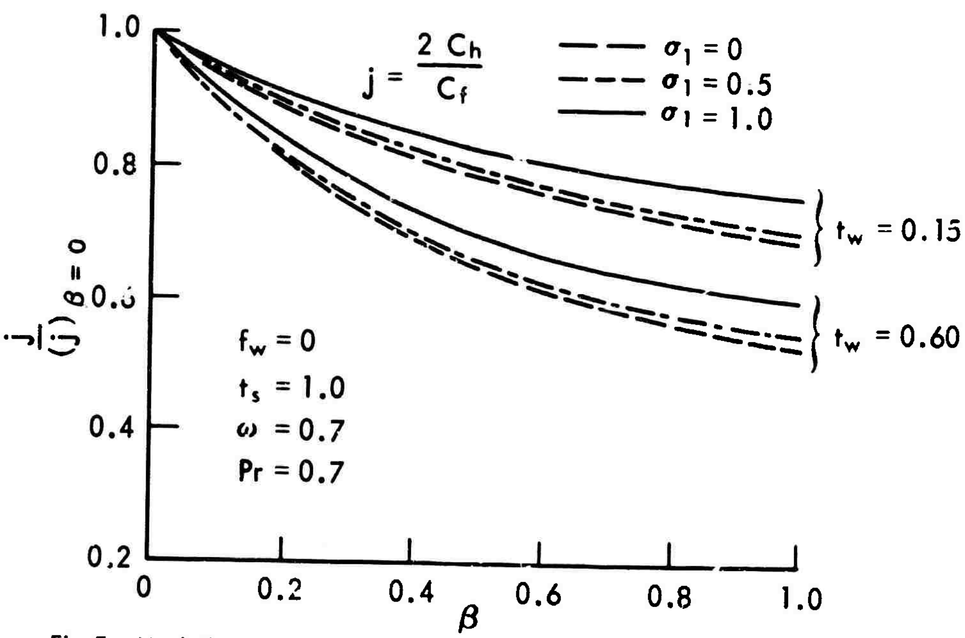

Fig.7—Variation of the Reynolds analogy function with pressure gradient

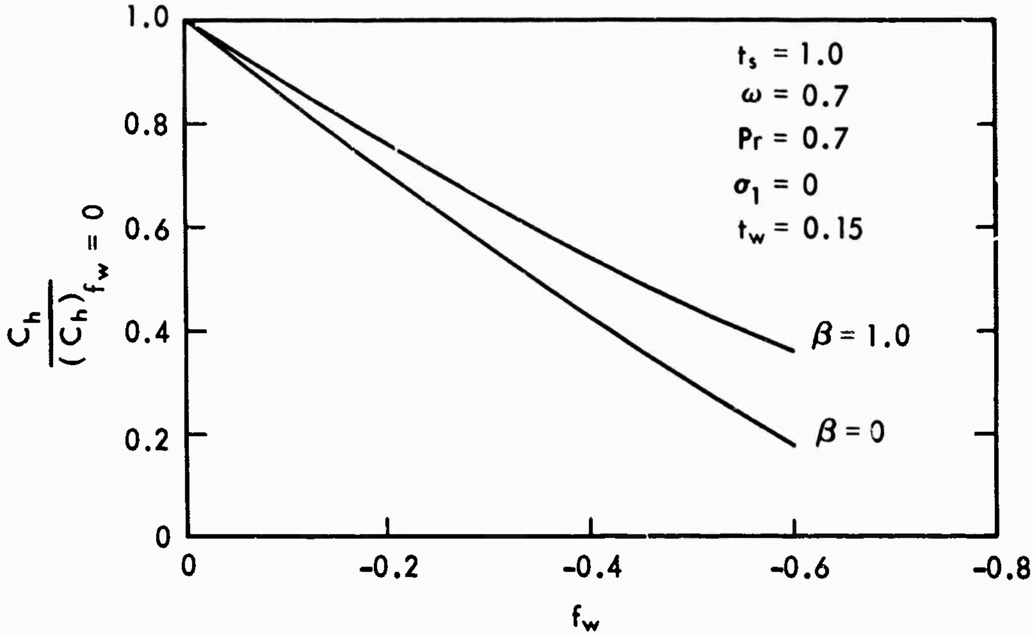

Fig.8—Variation of the heat transfer coefficient with mass transfer

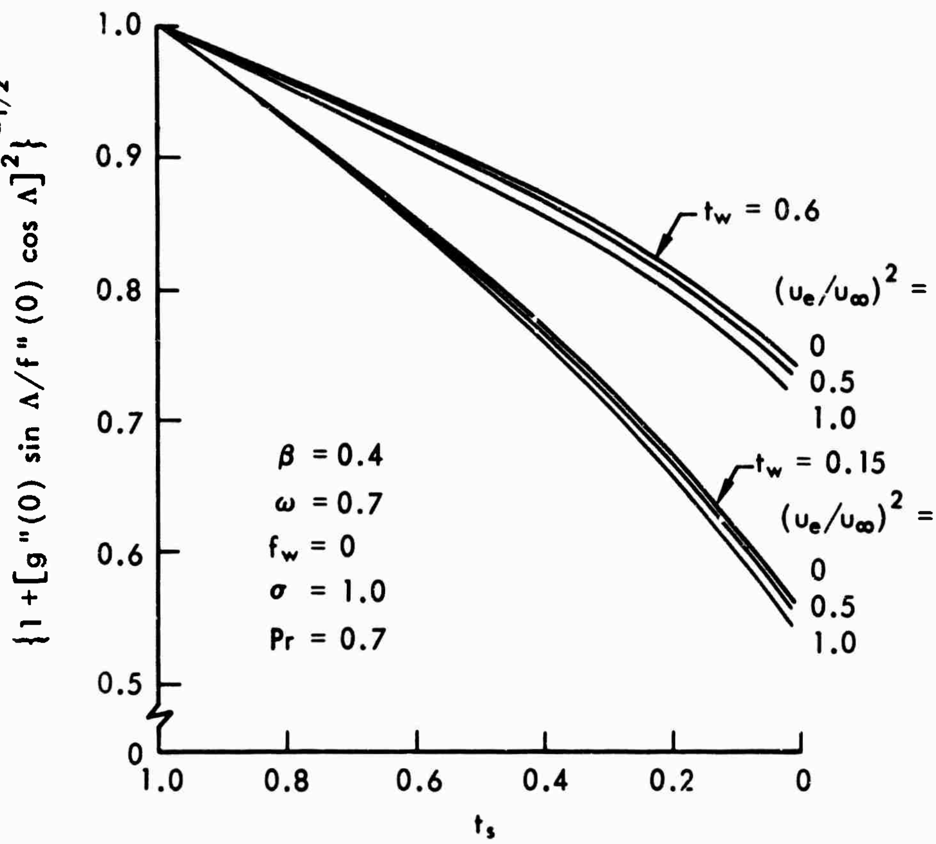

Fig.9—Variation of skin friction coefficient with sweep angle

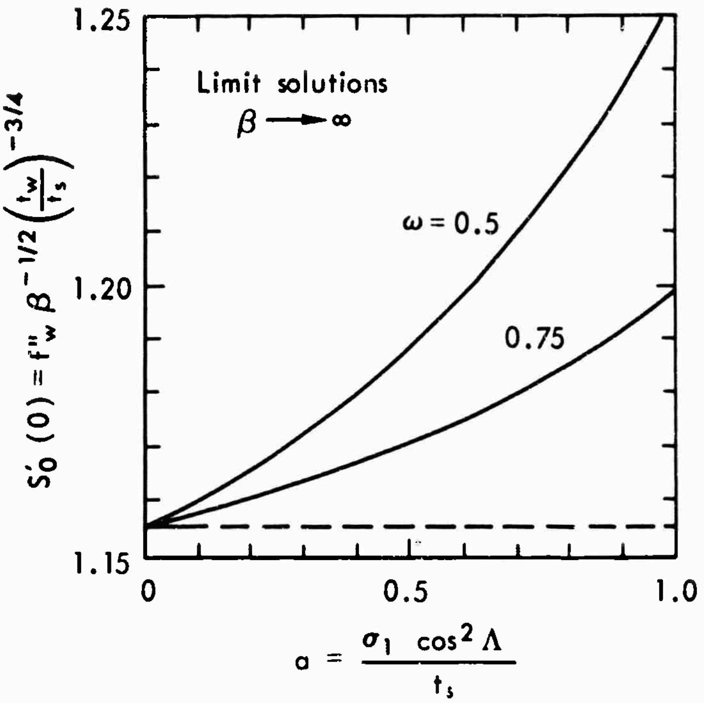

Fig. 10—Inner solutions for $\beta \rightarrow \infty$

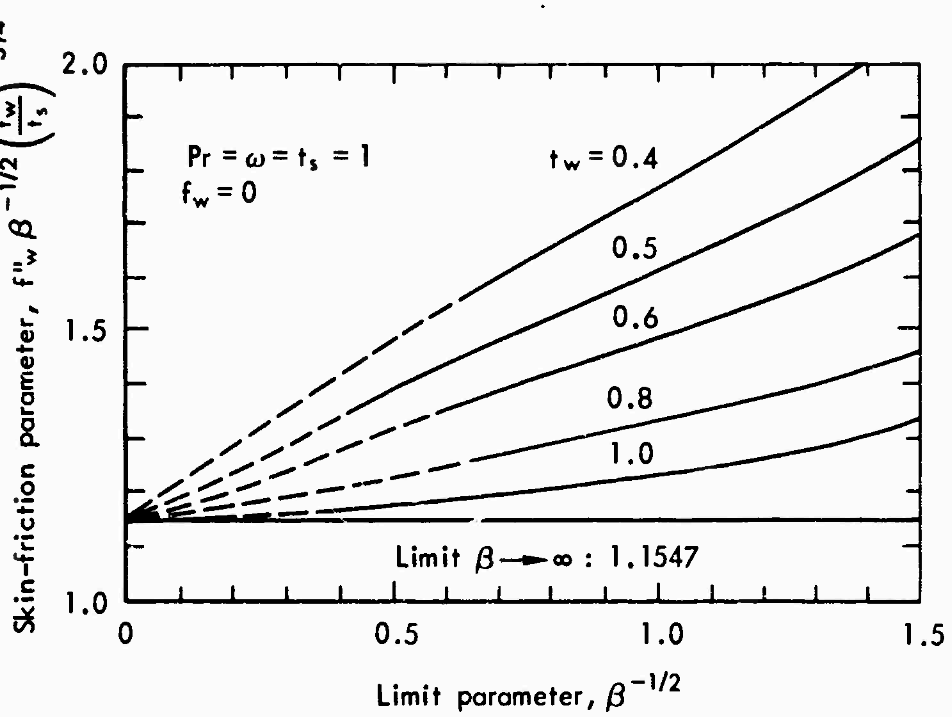

Fig. 11—Variation of skin friction with wall temperature for large β

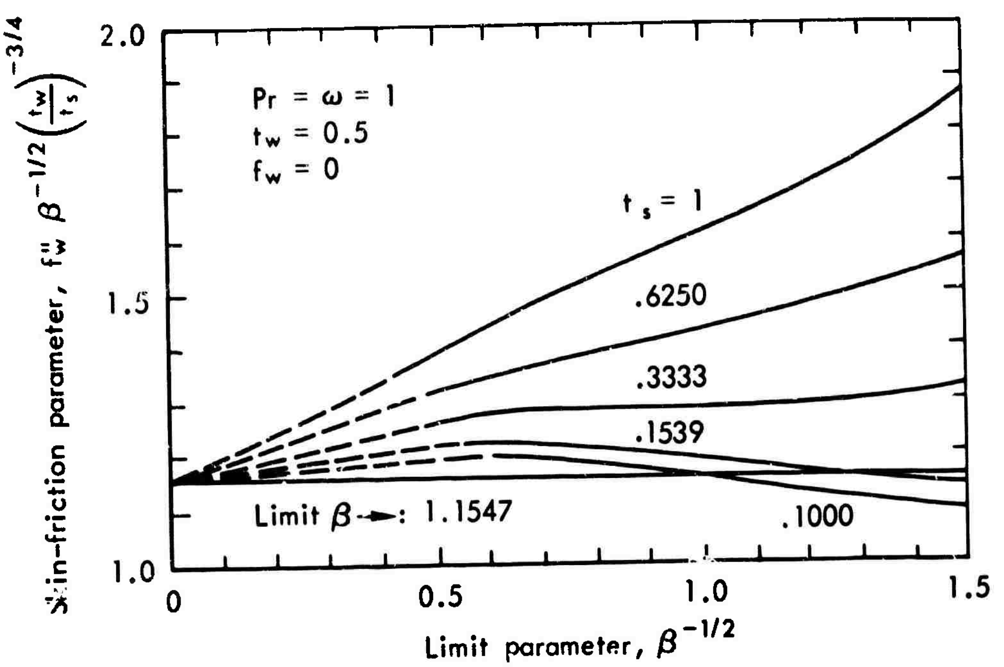

Fig. 12—Variation of skin friction with t_s for large β

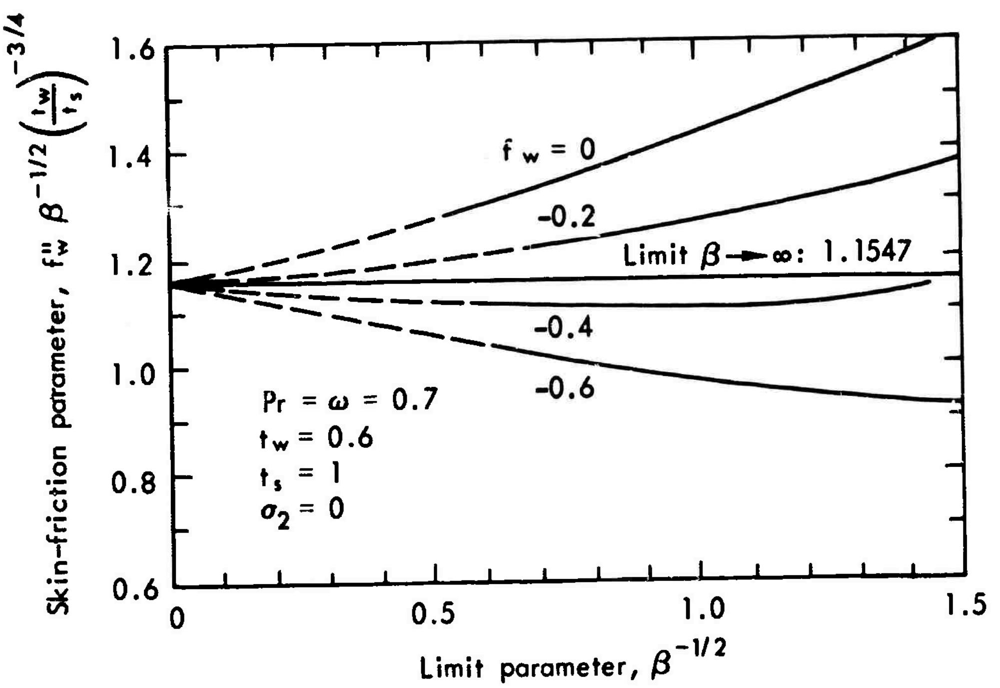

Fig. 13—Variation of skin friction with injection for large β

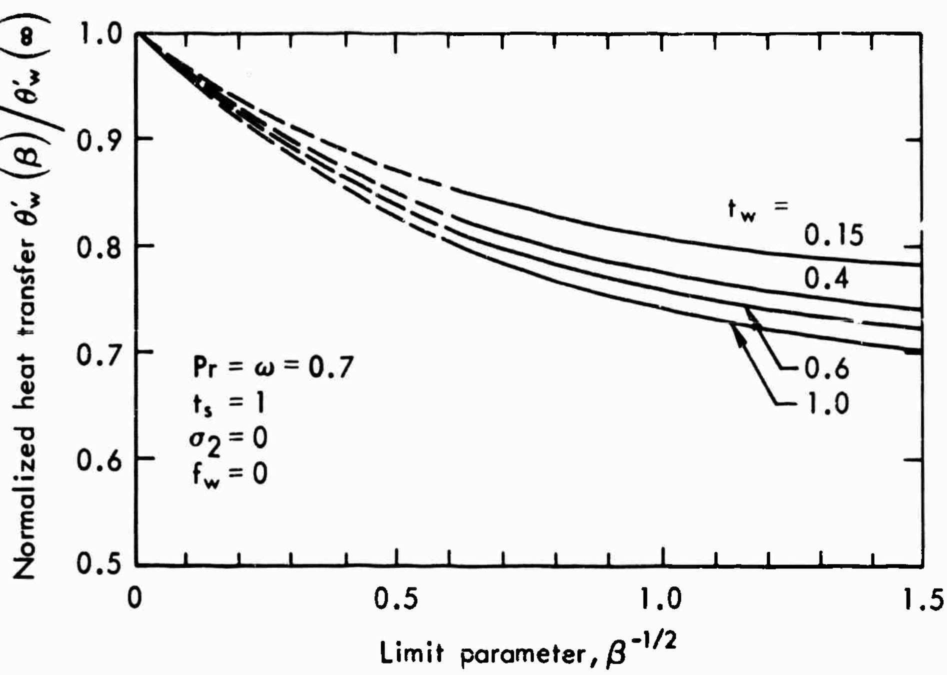

Fig. 14—Variation of heat transfer with wall temperature for large β

LIST OF TABLES

Table 1

SIMILAR SOLUTIONS FOR ω = Pr = 1

This table is summarized on pages 26, 27.

β	f_w	t_s	t_w	$f''(0)$	$\theta'(0) = g'(0)$	I_1	I_2	$I_1(1)$	$I_1(2)$	$I_1(3)$
0	0	1	0	0.4696	0.4696	0.4696	0.4696	1.686	1.686	1.217
			1.0	0.4696	0.4696	1.686	0.4696	1.686	1.686	1.217
	-0.1414	1	0	0.3700	0.3700	0.5114	0.5114	1.896	1.896	1.385
			1.0	0.3700	0.3700	1.896	0.5114	1.896	1.896	1.385
	-0.2828	1	0	0.2766	0.2766	0.5594	0.5594	2.163	2.163	1.603
			1.0	0.2766	0.2766	2.163	0.5594	2.163	2.163	1.603
	-0.4243	1	0	0.1907	0.1907	0.6150	0.6150	2.516	2.516	1.901
			1.0	0.1907	0.1907	2.516	0.6150	2.516	2.516	1.901
	-0.5657	1	0	0.1143	0.1143	0.6800	0.6800	3.021	3.021	2.341
			1.0	0.1143	0.1143	3.021	0.6800	3.021	3.021	2.341
	-0.7071	1	0	0.0502	0.0502	0.7573	0.7573	3.862	3.862	3.105
			1.0	0.0502	0.0502	3.862	0.7573	3.862	3.862	3.105
	-0.7782	1	0	0.0243	0.0243	0.8025	0.8025	4.628	4.628	3.825
			1.0	0.0243	0.0243	4.628	0.802	4.628	4.628	3.825
	-0.8485	1	0	0.0048	0.0048	0.8533	0.8533	6.403	6.403	5.550
			1.0	0.0048	0.0048	6.404	0.8533	6.403	6.403	5.550
	-0.8755	1	0	0.0003	0.0003	0.8621	0.8621	9.591	9.591	8.729
			1.0	0.0003	0.0003	9.591	0.8621	9.591	9.591	8.729

Table 1, cont.

β	f_w	t_s	t_w	$f''(0)$	$\theta'(0) = g'(0)$	I_1	I_2	$I_1(1)$	$I_1(2)$	$I_1(3)$
0.05	0	1	0	0.4848	0.4733	0.4453	0.4626	1.675	1.654	1.208
			0.5	0.5082	0.4773	1.023	0.4570	1.663	1.623	1.199
			1.0	0.5311	0.4812	1.593	0.4514	1.652	1.593	1.191
0.1	0	1	0.15	0.5123	0.4788	0.5910	0.4532	1.658	1.607	1.196
			0.4	0.5347	0.4825	0.8668	0.4479	1.647	1.579	1.188
			0.6	0.5523	0.4854	1.085	0.4439	1.639	1.558	1.182
			1.0	0.5870	0.4909	1.516	0.4354	1.624	1.516	1.176
0.2	0	0.1539	0	0.7839	0.5[illegible]49	0.3600	0.3158	1.503	1.101	1.081
			0.5	1.222	0.5845	0.7895	0.1958	1.393	0.7174	0.9999
			1.0	1.611	0.6221	1.180	0.0777	1.320	0.4077	0.9456
		0.3333	0	0.6238	0.5039	0.3761	0.3981	1.585	1.383	1.142
			0.5	0.8506	0.5344	0.8479	0.3418	1.508	1.154	1.084
			1.0	.058	0.5593	1.287	0.2855	1.450	0.9607	1.042
		0.625	0	0.5543	0.4891	0.3842	0.4314	1.627	1.515	1.173
			0.5	0.6836	0.5082	0.8817	0.4014	1.576	1.373	1.134
			1.0	0.8047	0.5248	1.355	0.3712	1.533	1.247	1.103
		1	0	0.5233	0.4821	0.3881	0.4457	1.648	1.575	1.188
			0.5	0.6070	0.4951	0.8995	0.4271	1.612	1.480	1.161
			1.0	0.6867	0.5069	1.392	0.4082	1.580	1.392	1.138
0.25	0	0.1	0	1.017	0.5737	0.3304	0.1905	1.412	0.7389	1.014
			0.5	1.756	0.6425	0.7123	-0.0248	1.280	0.1840	0.9169
			1.0	2.401	0.6921	1.054	-0.2332	1.199	-0.2584	0.8573

Table 1, cont.

β	f_w	t_s	t_w	f"(0)	θ'(0) = g'(0)	I_1	I_2	$I_1(1)$	$I_1(2)$	$I_1(3)$
0.2857	0	1	0	0.5419	0.4862	0.3629	0.4382	1.636	1.542	1.179
			0.2	0.5883	0.4932	0.5593	0.4285	1.617	1.491	1.165
			0.4	0.6334	0.4998	0.7517	0.4186	1.599	1.443	1.151
			0.6	0.6774	0.5061	0.9406	0.4087	1.582	1.396	1.139
			0.8	0.7205	0.5121	.126	0.3987	1.566	1.352	1.127
			1.0	0.7627	0.5179	.309	0.3887	1.551	1.309	1.116
	-0.2	1	0	0.4032	0.3511	0.3929	0.4909	1.910	1.793	1.400
			0.2	0.4547	0.3604	0.6183	0.4781	1.875	1.716	1.373
			0.4	0.5042	0.3690	0.8367	0.4651	1.843	1.646	1.348
			0.6	0.5520	0.3769	1.049	0.4522	1.815	1.580	1.327
			0.8	0.5983	0.3843	1.257	0.4392	1.789	1.519	1.307
			1.0	0.6434	0.3913	1.460	0.4262	1.766	1.460	1.289
	-0.4	1	0	0.2758	0.2301	0.4252	0.5543	2.279	2.133	1.707
			0.2	0.3335	0.2425	0.6877	0.5370	2.209	2.009	1.651
			0.4	0.3876	0.2534	0.9379	0.5197	2.151	1.901	1.605
			0.6	0.4392	0.2631	1.178	0.5025	2.102	1.805	1.566
			0.8	0.4886	0.2720	1.411	0.4854	2.059	1.717	1.532
			1.0	0.5362	0.2802	1.637	0.4684	2.021	1.637	1.501
	-0.6	1	0	0.1629	0.1273	0.4591	0.6318	2.810	2.628	2.169
			0.2	0.2279	0.1436	0.7712	0.6076	2.656	2.402	2.038
			0.4	0.2867	0.1568	1.060	0.5839	2.545	2.227	1.945
			0.6	0.3414	0.1682	1.332	0.5608	2.457	2.081	1.872
			0.8	0.3931	0.1782	1.593	0.5381	2.385	1.955	1.812
			1.0	0.4426	0.1873	1.843	0.5159	2.324	1.843	1.762

Table 1, cont.

β	f_w	t_s	t_w	$f''(0)$	$\theta'(0) = g'(0)$	I_1	I_2	$I_1(1)$	$I_1(2)$	$I_1(3)$
0.3	0	1	0	0.5448	0.4868	0.3591	0.4371	1.634	1.537	1.178
			0.2	0.5931	0.4941	0.5539	0.4270	1.614	1.484	1.163
			0.4	0.6402	0.5009	[illegible]	0.4168	1.596	1.434	1.149
			0.6	0.6860	0.5074	0.9318	0.4065	1.578	1.386	1.136
			0.8	0.7309	0.5136	1.116	0.3961	1.562	1.341	1.124
			1.0	0.7748	0.5195	1.297	0.3857	1.547	1.297	1.113
			2.0	0.9829	0.5457	2.165	0.3334	1.484	1.099	1.066
0.4	0	1	0	0.5639	0.4908	0.3350	0.4299	1.623	1.505	1.170
			0.2	0.6254	0.4997	0.5198	0.4175	1.599	1.441	1.152
			0.4	0.6850	0.5079	0.7001	0.4050	1.577	1.381	1.135
			0.6	0.7429	0.5157	0.8765	0.3923	1.557	1.325	1.120
			0.8	0.7993	0.5231	1.049	0.3795	1.538	1.271	1.107
			1.0	0.8544	0.5300	1.219	0.3667	1.521	1.219	1.094
	-0.2	1	0	0.4246	0.3565	0.3591	0.4809	1.889	1.743	1.384
			0.2	0.4924	0.3681	0.5689	0.4648	1.846	1.649	1.351
			0.4	0.5571	0.3785	0.7716	0.4485	1.809	1.565	1.322
			0.6	0.6194	0.3881	0.9680	0.4322	1.776	1.487	1.297
			0.8	0.6795	0.3970	1.159	0.4157	1.747	1.414	1.275
			1.0	0.7378	0.4052	1.346	0.3993	1.720	1.346	1.254
	-0.4	1	0	0.2959	0.2363	0.3842	0.5423	2.242	2.062	1.678
			0.4	0.4413	0.2643	0.8547	0.4994	2.096	1.791	1.561
			0.6	0.5077	0.2757	1.074	0.4780	2.042	1.681	1.518
			0.8	0.5711	0.2861	1.286	0.4567	1.995	1.582	1.480
			1.0	0.6321	0.2956	[illegible]	0.4357	1.954	1.491	1.448

Table 1, cont.

β	f_w	t_s	t_w	f"(0)	θ'(0) = g'(0)	I_1	I_2	$I_1(1)$	$I_1(2)$	$I_1(3)$
0.4	-0.6	1	0	0.1809	0.1340	0.4094	0.6171	2.741	2.520	2.111
			0.2	0.2648	0.1532	0.6923	0.5878	2.572	2.267	1.968
			0.4	0.3400	0.1686	0.9524	0.5590	2.453	2.074	1.869
			0.6	0.4097	0.1816	1.197	0.5307	2.362	1.915	1.793
			0.8	0.4756	0.1931	1.431	0.5031	2.288	1.778	1.732
			1.0	0.5384	0.2034	1.656	0.4760	2.225	1.656	1.681
0.5	0	0.1	0	1.365	0.6211	0.2704	0.0133	1.315	0.2987	0.9427
			0.5	2.600	0.7113	0.5882	-0.3417	1.169	-0.4635	0.8355
			1.0	3.661	0.7739	0.8688	-0.6827	1.085	-1.075	0.7738
		0.1539	0	1.089	0.5828	0.2827	0.1703	1.393	0.6761	1.000
			0.5	1.966	0.6577	0.6257	-0.0671	1.256	0.0812	0.8985
			1.0	2.728	0.7111	0.9314	-0.2988	1.172	-0.3942	0.8374
		0.3333	0	0.7812	0.5328	0.3000	0.3311	1.509	1.138	1.086
			0.5	1.248	0.5833	0.6856	0.2195	1.398	0.7649	1.003
			1.0	1.662	0.6215	1.037	0.1073	1.324	0.4623	0.9480
		0.625	0	0.6438	0.5070	0.3097	0.3961	1.577	1.366	1.136
			0.5	0.9167	0.5410	0.7244	0.3371	1.494	1.122	1.074
			1.0	1.165	0.5684	1.110	0.2769	1.432	0.9165	1.028
		1	0	0.5811	0.4942	0.3146	0.4238	1.613	1.477	1.162
			0.15	0.6368	0.5020	0.4474	0.4131	1.592	1.422	1.147
			0.2	0.6550	0.5045	0.4909	0.4095	1.586	1.404	1.142
			0.4	0.7262	0.5140	0.6625	0.3949	1.561	1.337	1.124
			0.5	0.7609	0.5185	0.7469	0.3876	1.550	1.304	1.115
			0.6	0.7952	0.5228	0.8301	0.3802	1.539	1.273	1.107
			0.8	0.8623	0.5311	0.9941	0.3653	1.519	1.212	1.092
			1.0	0.9277	0.5390	1.155	0.3503	1.500	1.155	1.078
			2.0	1.235	0.5729	1.920	0.2746	1.424	0.8988	1.022

Table 1, cont.

β	f_w	t_s	t_w	$f''(0)$	$\theta'(0) = g'(0)$	I_1	I_2	$I_1(1)$	$I_1(2)$	$I_1(3)$
0.5	-0.5	0.1539	0	0.6921	0.2956	0.3132	0.1742	1.846	0.8092	1.373
			0.5	1.651	0.3896	0.7224	-0.1967	1.550	-0.1245	1.140
		0.3333	0	0.4265	0.2377	0.3394	0.4173	2.117	1.548	1.588
			0.5	0.9556	0.3081	0.8202	0.2252	1.814	0.8506	1.346
			1.0	1.395	0.3540	1.233	0.0458	1.660	0.3782	1.225
		0.625	0	0.3067	0.2049	0.3560	0.5219	2.312	1.974	1.744
			0.5	0.6290	0.2581	0.8954	0.4127	2.028	1.428	1.515
			1.0	0.9003	0.2941	1.365	0.3081	1.874	1.060	1.392
		1	0	0.2512	0.1873	0.3654	0.5685	2.433	2.209	1.843
			0.5	0.4714	0.2290	0.9464	0.4981	2.179	1.764	1.635
			1.0	0.6594	0.2583	1.460	0.4294	2.031	1.460	1.517
0.75	0	0.1	0	1.632	0.6515	0.2329	-0.1147	1.260	0.0149	0.9023
			0.5	3.278	0.7549	0.5136	-0.5742	1.109	-0.886[illegible]	0.7914
			1.0	4.685	0.8254	0.7599	-1.015	1.024	-1.617	0.7294
		0.3333	0	0.8796	0.5485	0.2605	0.2935	1.472	1.011	1.058
			0.5	1.509	0.6089	0.6046	0.1487	1.347	0.5673	0.9655
			1.0	2.062	0.6534	0.9152	0.0030	1.268	0.2101	0.9067
		1	0	0.6181	0.5012	0.2748	0.4120	1.594	1.423	1.148
			0.15	0.6948	0.5112	0.3953	0.3984	1.568	1.355	1.129
			0.4	0.8173	0.5262	0.5896	0.3751	1.531	1.250	1.101
			0.5	0.8648	0.5318	0.6654	0.3656	1.517	1.211	1.091
			0.6	0.9112	0.5371	0.7402	0.3560	1.504	1.173	1.081
			1.0	1.090	0.5568	1.031	0.3174	1.460	1.031	1.048

Table 1, cont.

β	f_w	t_s	t_w	$f''(0)$	$\theta'(0) = g'(0)$	I_1	I_2	$I_1(1)$	$I_1(2)$	$I_1(3)$
1	0	0.1	0	1.850	0.6735	0.2063	-0.2130	1.223	-0.1898	0.8754
			0.5	3.859	0.7865	0.4615	-0.7556	1.069	-1.197	0.7624
			1.0	5.567	0.8625	0.6842	-1.275	0.9846	-2.019	0.7007
		0.1539	0	1.431	0.6253	0.2166	0.0235	1.309	0.3071	0.9382
			0.5	2.866	0.7212	0.4934	-0.3404	1.158	-0.4747	0.8270
			1.0	4.094	0.7870	0.7370	-0.6945	1.072	-1.105	0.7640
			2.0	6.254	0.8817	1.173	-1.370	0.9673	-2.164	0.6879
		0.25	0	1.102	0.5815	0.2267	0.1957	1.398	0.7265	1.003
			0.5	2.079	0.6593	0.5277	-0.0320	1.256	0.1385	0.8987
			1.0	2.923	0.7143	0.7955	-0.2587	1.172	-0.3327	0.8363
		0.3333	0	0.9607	0.5603	0.2318	0.2652	1.445	0.9194	1.038
			0.5	1.734	0.6280	0.5466	0.0942	1.312	0.4243	0.9396
			1.0	2.409	0.6770	0.8289	-0.0781	1.230	0.0271	0.8787
		0.5	0	0.8105	0.5358	0.2379	0.33[illegible]6	1.503	1.135	1.081
			0.5	1.364	0.5904	0.5711	0.2218	1.386	0.7498	0.9938
			1.0	1.853	0.6313	0.8735	0.1061	1.309	0.4382	0.9367
			2.0	2.726	0.6931	1.425	-0.1244	1.206	-0.0782	0.8613
		0.625	0	0.7475	0.5248	0.2408	0.3622	1.531	1.229	1.102
			0.5	1.206	0.5728	0.5833	0.2730	1.424	0.8962	1.021
			1.0	1.615	0.6094	0.8965	0.1805	1.350	0.6242	0.9671
		0.8333	0	0.6823	0.5130	0.2439	0.3897	1.562	1.330	1.124
			0.5	1.041	0.5530	0.5975	0.3239	1.468	1.056	1.054
			1.0	1.364	0.5845	0.9240	0.2551	1.401	0.8287	1.004

Table 1, cont.

β	f_w	t_s	t_w	f"(0)	θ'(0) = g'(0)	I_1	I_2	$I_1(1)$	$I_1(2)$	$I_1(3)$
1	0	1	0	0.6489	0.5067	0.2456	0.4033	1.579	1.383	1.137
			0.15	0.7445	0.5183	0.3570	0.3875	1.550	1.305	1.115
			0.2	0.7755	0.5219	0.3934	0.3821	1.541	1.280	1.109
			0.4	0.8963	0.5357	0.5360	0.3603	1.508	1.186	1.084
			0.5	0.9548	0.5421	0.6056	0.3491	1.493	1.142	1.073
			0.6	1.012	0.5482	0.6743	0.3380	1.479	1.099	1.063
			0.8	1.124	0.5597	0.8089	0.3153	1.454	1.018	1.043
			1.0	1.233	0.5705	0.9402	0.2923	1.430	0.9402	1.026
			2.0	1.737	0.6156	1.560	0.1760	1.340	0.6010	0.9595
	-0.5	0.1539	0	0.9717	0.3391	0.2288	-0.0152	1.690	0.3219	1.250
			0.5	2.512	0.4536	0.5475	-0.5454	1.399	-0.8079	1.023
			1.0	3.774	0.5[illegible]9	0.8164	-1.030	1.262	-1.635	0.9188
		0.3333	0	0.5719	0.2678	0.2480	0.3277	1.970	1.215	1.471
			0.5	1.426	0.3558	0.6236	0.0549	1.657	0.3895	1.222
			1.0	2.129	0.4115	0.9434	-0.2016	1.505	-0.1807	1.104
		0.625	0	0.3910	0.2261	0.2603	0.4744	2.186	1.732	1.642
			0.5	0.9146	0.2945	0.6837	0.3208	1.875	1.083	1.392
			1.0	1.349	0.3390	1.048	0.1719	1.717	0.6471	1.268
		1	0	0.3067	0.2031	0.2675	0.5392	2.327	2.024	1.756
			0.5	0.6669	0.2580	0.7255	0.4414	2.035	1.485	1.519
			1.0	0.9692	0.2950	1.125	0.3441	1.878	1.125	1.394
	-1.0	0.1539	0	0.5068	0.1298	0.2363	-0.0286	2.277	0.4951	1.766
			0.5	2.144	0.2520	0.6063	-0.7956	1.685	-1.199	1.270
			1.0	3.416	0.3212	0.9018	-1.444	1.478	-2.267	1.103

Table 1, cont.

β	f_w	t_s	t_w	$f''(0)$	$\theta'(0) = g'(0)$	I_1	I_2	$I_1(1)$	$I_1(2)$	$I_1(3)$
1	-1.0	0.3333	0	0.2159	0.0699	0.2570	0.4449	2.918	1.892	2.319
			0.5	1.139	0.1652	0.7202	0.0174	2.091	0.3794	1.601*
			1.0	1.847	0.2184	1.071	-0.3663	1.830	-0.4481	1.385
		0.625	0	0.0970	0.0369	0.2701	0.6648	3.604	2.963	2.934
			0.5	0.6740	0.1126	0.8053	0.3855	2.484	1.343	1.931
			1.0	1.112	0.1546	1.225	0.1513	2.170	0.6582	1.665
		1	0	0.0481	0.0202	0.2780	0.7624	4.234	3.801	3.523
			0.5	0.4519	0.0822	0.8805	0.5713	2.817	1.989	2.218
			1.0	0.7566	0.1168	1.351	0.4058	2.459	1.351	1.908
1.4	0	0.1	0	2.135	0.6990	0.1762	-0.3318	1.183	-0.4251	0.8463
			0.5	4.657	0.8234	0.4025	-0.9785	1.027	-1.561	0.7316
			1.0	6.790	0.9059	0.5991	-1.596	0.9429	-2.495	0.6703*
1.5	0	0.1539	0	1.677	0.6507	0.1791	-0.0687	1.264	0.0950	0.9053
			0.5	3.566	0.7591	0.4190	-0.5174	1.108	-0.8020	0.7901
			1.0	5.172	0.8321	0.6286	-0.9538	1.021	-1.531	0.7268
		0.3333	0	1.091	0.5772	0.1924	0.2249	1.409	0.7930	1.011
			0.5	2.115	0.6555	0.4668	0.0144	1.266	0.2264	0.9051
			1.0	3.000	0.7107	0.7107	-0.1984	1.180	-0.2279	0.8419*
		0.625	0	0.8233	0.5362	0.2004	0.3422	1.504	1.148	1.081
			0.5	1.435	0.5928	0.5008	0.2330	1.383	0.7644	0.9915
			1.0	1.974	0.6350	0.7728	0.1190	1.305	0.4535	0.9334

*Convergence to 10^{-4}.

Table 1, cont.

β	f_w	t_s	t_w	f"(0)	θ'(0) = g'(0)	I_1	I_2	$I_1(1)$	$I_1(2)$	$I_1(3)$
1.5	0	1	0	0.6987	0.5147	0.2049	0.3914	1.558	1.326	1.122
			0.15	0.8278	0.52[illegible]	0.3035	0.3725	1.524	1.235	1.095
			0.2	0.8695	0.533[illegible]	0.3356	0.3661	1.514	1.206	1.088
			0.4	1.031	0.5498	0.4610	0.3395	1.476	1.097	1.060
			0.5	1.109	0.5574	0.5220	0.3259	1.460	1.046	1.048
			0.6	1.185	0.5646	0.5821	0.3122	1.444	0.9966	1.036
			0.8	1.334	0.5781	0.6996	0.2844	1.416	0.9026	1.015
			1.0	1.477	0.5906	0.8141	0.2562	1.390	0.8141	0.9963
			2.0	2.140	0.6423	1.352	0.1128	1.294	0.4268	0.9251*
1.8	0	0.1	0	2.368	0.7176	0.1549	-0.4203	1.156	-0.5935	0.8266
			0.5	5.345	0.8504	0.3623	-1.140	0.9987	-1.811	0.7107
			1.0	7.854	0.9378	0.5387	-1.842	0.9148	-2.847	0.6498*
2	0	0.1	0	2.471	0.7252	0.1464	-0.4568	1.146	-0.6615	0.8188
			0.5	5.660	0.8616	0.3439	-1.218	0.9875	-1.936	0.7026
			1.0	8.342	0.9509	0.5144	-1.945	0.9039	-2.991	0.6418*
		0.1539	0	1.871	0.6681	0.1542	-0.1330	1.236	-0.0466	0.8842
			0.5	4.156	0.7855	0.3694	-0.6442	1.076	-1.025	0.7666
			1.0	6.088	0.8635	0.5563	-1.141	0.9893	-1.824	0.7034
		0.3333	0	1.194	0.5891	0.1661	0.1072	1.385	0.7086	0.9932
			0.5	2.437	0.6749	0.4132	-0.0423	1.235	0.0925	0.8825
			1.0	3.504	0.7345	0.6315	-0.2848	1.148	-0.4016	0.8183*
		0.625	0	0.8837	0.5444	0.1734	0.3287	1.485	1.094	1.067
			0.5	1.629	0.6073	0.4505	0.2095	1.356	0.6839	0.9713*
			1.0	2.281	0.6533	0.6892	0.0752	1.275	0.3377	0.9111

*Convergence to 10^{-4}.

Table 1, cont.

β	f_w	t_s	t_w	$f''(0)$	$\theta'(0) = g'(0)$	I_1	I_2	$I_1(1)$	$I_1(2)$	$I_1(3)$
2	0	1	0	0.7386	0.5206	0.1775	0.3837	1.544	1.288	1.111
			0.15	0.8972	0.5367	0.2673	0.3626	1.506	1.187	1.082
			0.2	0.9483	0.5417	0.2965	0.3553	1.495	1.156	1.074
			0.4	1.146	0.5601	0.4101	0.3254	1.454	1.036	1.044
			0.5	1.241	0.5686	0.4653	0.3101	1.436	0.9804	1.030
			0.6	1.333	0.5766	0.5194	0.2945	1.420	0.9266	1.018
			0.8	1.513	0.5915	0.6254	0.2629	1.390	0.8245	0.9957*
			1.0	1.687	0.6052	0.7284	0.2309	1.363	0.7284	0.9760*
			2.0	2.488	0.6615	1.193	0.0587	1.263	0.2908	0.9022*
2.4	0	0.1539	0	2.001	0.6789	0.1394	-0.1722	1.219	-0.1309	0.8718
			0.5	4.574	0.8018	0.3462	-0.7022	1.058	-1.119	0.7529*
			1.0	6.741	0.8830	0.5131	-1.258	0.9708	-2.003	0.6898**
		0.3333	0	1.264	0.5965	0.1504	0.1805	1.370	0.6582	0.9826
			0.5	2.666	0.6871	0.3906	-0.0631	1.217	0.0406	0.8691*
			1.0	3.864	0.7493	0.5838	-0.3389	1.129	-0.5072	0.8044*
		0.625	0	0.9248	0.5495	0.1573	0.3208	1.473	1.061	1.059
			0.5	1.767	0.6164	0.4113	0.1881	1.340	0.6213	0.9593
			1.0	2.501	0.6648	0.6387	0.0481	1.257	0.2678	0.8978*
		1	0	0.7659	0.5244	0.1611	0.3791	1.535	1.265	1.104
			0.5	1.335	0.5757	0.7309	0.3005	1.422	0.9408	1.020*
			1.0	1.838	0.6145	0.6762	0.2151	1.347	0.6762	0.9637

*Convergence to 10^{-4}.

**Convergence to 10^{-3}.

Table 1, cont.

β	f_w	t_s	t_w	$f''(0)$	$\theta'(0) = g'(0)$	I_1	I_2	$I_1(1)$	$I_1(2)$	$I_1(3)$
2.8	0	0.1	0	2.816	0.7484	0.1209	-0.5687	1.115	-0.8655	0.7961
			0.5	6.774	0.8961	0.2933	-1.439	0.9550	-2.268	0.6787
			1.0	10.083	0.9961	0.4412	-2.268	0.8721	-3.436	0.6186**
		0.1539	0	2.117	0.6877	0.1443	-0.1507	1.206	-0.0900	0.8619*
			1.0	7.342	0.8991	0.4784	-1.355	0.9562	-2.148	0.6791**
		0.3333	0	1.325	0.6026	0.1379	0.1671	1.359	0.6179	0.9739
			1.0	4.196	0.7617	0.5460	-0.3826	1.114	-0.5904	0.7933**
		0.625	0	0.9612	0.5538	0.1444	0.3144	1.464	1.035	1.052
			0.5	1.895	0.6241	0.3874	0.1741	1.327	0.5778	0.9494
			1.0	2.704	0.6744	0.5979	0.0256	1.243	0.2111	0.8870*
		1	0	0.7901	0.5275	0.1480	0.3756	1.527	1.246	1.098
			0.5	1.421	0.5817	0.4033	0.2928	1.411	0.9088	1.011**
			1.0	1.978	0.6223	0.6341	0.2023	1.333	0.6341	0.9537
3.4	0	1	0.5	1.541	0.5893	0.3697	0.2835	1.396	0.8699	1.000
			1.0	2.170	0.6320	0.5929	0.1919	1.317	0.5938	0.9416**
4	0	1	0	0.8502	0.5346	0.1205	0.3684	1.511	1.207	1.086
			0.5	1.650	0.5955	0.3434	0.2762	1.385	0.8393	0.9918
			1.0	2.347	0.6401	0.5447	0.1751	1.305	0.5447	0.9320**
5	0	1	0	0.8907	0.5389	0.1052	0.3647	1.502	1.184	1.079
			0.5	1.815	0.6042	0.3096	0.2668	1.368	0.7994	0.9796
			1.0	2.616	0.6509	0.4923	0.1586	1.288	0.4923	0.9196**

*Convergence to 10^{-4}.

**Convergence to 10^{-3}.

Table 2

SIMILAR SOLUTIONS FOR A POWER-LAW VISCOSITY RELATION, Pr = 0.7, f_w = 0

This table is summarized on pages 28--30.

β	ω	t_s	σ	$\left(\frac{u_e}{u_\infty}\right)^2$	t_w	$f''(0)$	$\theta'(0)$	$g'(0)$	I_1	I_2	$I_1(1)$	$I_1(2)$	$I_1(3)$
0	0.5	all	0	-	0.15	0.3490	0.3034	0.3490	0.4037	0.3490	1.411	1.411	1.185
					0.4	0.4143	0.3625	0.4143	0.7886	0.4144	1.571	1.571	1.303
					0.6	0.4399	0.3861	0.4399	1.089	0.4399	1.626	1.626	1.343
					1.0	0.4696	0.4139	0.4696	1.686	0.4696	1.686	1.686	1.385
			0.5	-	0.15	0.3734	0.2963	0.3734	0.4608	0.3735	1.452	1.452	1.166
					0.4	0.4394	0.3341	0.4394	0.8600	0.4394	1.610	1.610	1.250
					0.6	0.4642	0.3246	0.4642	1.167	0.4642	1.663	1.663	1.240
					0.9151	0.4876	0.0	0.4876	1.645	0.4876	1.710	1.710	0.7665
			1	-	0.05	0.3507	0.2527	0.3507	0.3514	0.3494	1.304	1.304	1.003
					0.15	0.4328	0.3026	0.4328	0.5519	0.4303	1.523	1.523	1.142
					0.4	0.4987	0.3045	0.4987	0.9513	0.4924	1.672	1.672	1.201
					0.6	0.5208	0.2392	0.5208	1.251	0.5124	1.717	1.717	1.163
					0.8009	0.5343	0.0	0.5343	1.546	0.5239	1.742	1.742	0.9846
					1.1	0.5491	1.521	0.5491	1.953	0.5301	1.753	1.753	2.000
	0.7	all	0	-	0.15	0.3920	0.3428	0.3920	0.4414	0.3921	1.508	1.508	1.255
					0.4	0.4353	0.3820	0.4353	0.8137	0.4353	1.614	1.614	1.334
					0.6	0.4514	0.3969	0.4514	1.106	0.4514	1.649	1.649	1.359
					1.0	0.4696	0.4139	0.4696	1.686	0.4696	1.686	1.686	1.385
			0.5	-	0.15	0.4089	0.3250	0.4089	0.4970	0.4089	1.538	1.537	1.224
					0.4	0.4512	0.3431	0.4512	0.8778	0.4512	1.640	1.640	1.270
					0.6	0.4663	0.3263	0.4663	1.173	0.4663	1.672	1.672	1.248
					0.9162	0.4802	0.0	0.4802	1.656	0.4802	1.700	1.700	0.7691

Table 2, cont.

β	ω	t_s	σ	$\left(\frac{u_e}{u_\infty}\right)^2$	t_w	$f''(0)$	$\theta'(0)$	$g'(0)$	I_1	I_2	$I_1(1)$	$I_1(2)$	$I_1(3)$
0	0.7	all	1	-	0.05	0.3924	0.2843	0.3924	0.3963	0.3923	1.442	1.442	1.100
					0.15	0.4451	0.3136	0.4451	0.5766	0.4448	1.586	1.585	1.188
					0.4	0.4835	0.3025	0.4835	0.9637	0.4832	1.681	1.681	1.195
					0.6	0.4960	0.2420	0.4960	1.253	0.4947	1.707	1.707	1.134
					0.8187	0.5034	0.0	0.5034	1.580	0.5029	1.725	1.725	0.8037
					1.1	0.5097	1.291	0.5097	1.982	0.5085	1.737	1.737	2.444
	1	all	0	-	0.15	0.4696	0.4139	0.4696	0.5091	0.4696	1.686	1.686	1.385
					0.4	0.4696	0.4139	0.4696	0.8554	0.4696	1.686	1.686	1.385
					0.6	0.4696	0.4139	0.4696	1.132	0.4696	1.686	1.686	1.385
					1.0	0.4696	0.4139	0.4696	1.686	0.4696	1.686	1.686	1.385
			0.5	-	0.15	0.4696	0.3739	0.4696	0.5595	0.4696	1.686	1.686	1.326
					0.4	0.4696	0.3572	0.4696	0.9057	0.4696	1.686	1.686	1.301
					0.6	0.4696	0.3281	0.4696	1.183	0.4696	1.686	1.686	1.259
					0.9179	0.4696	0.0	0.4696	1.623	0.4696	1.686	1.686	0.7732
			1	-	0.15	0.4696	0.3339	0.4696	0.6097	0.4696	1.686	1.686	1.267
					0.4	0.4696	0.3006	0.4696	0.9560	0.4696	1.686	1.686	1.217
					0.6	0.4696	0.2439	0.4696	1.233	0.4696	1.686	1.686	1.134
					0.8357	0.4696	0.0	0.4696	1.559	0.4696	1.686	1.686	0.7729
0.1	0.5	1	1		0.05	0.3[illegible]80	0.2597	0.3623	0.3187	0.3414	1.291	1.260	0.9912
					0.15	0.4733	0.3112	0.4476	0.5088	0.4197	1.513	1.464	1.124
					0.4	0.5055	0.3134	0.5194	0.8753	0.4717	1.652	1.575	1.166
					0.6	0.6066	0.2435	0.5450	1.14[illegible]	0.4840	1.690	1.591	1.106
					0.7948	0.6376	0.0	0.5613	1.409	0.4882	1.709	1.590	0.8825
					1.1	0.6789	1.644	0.5909	1.785	0.4852	1.709	1.566	2.185

Table 2, cont.

β	ω	t_s	σ	$\left(\frac{u_e}{u_\infty}\right)^2$	t_w	$f''(0)$	$\theta'(0)$	$g'(0)$	I_1	I_2	$I_1(1)$	$I_1(2)$	$I_1(3)$
0.1	0.7	0.3333	1	1	0.15	0.5523	0.3304	0.4188	0.5197	0.3964	1.531	1.387	1.134
					0.8158	0.7916	0.0	0.5580	1.375	0.3793	1.602	1.265	0.6223
		0.5	1	1	0.15	0.5191	0.3257	0.4643	0.5246	0.4142	1.550	1.452	1.149
					0.8140	0.7020	0.0	0.5435	1.399	0.4222	1.639	1.407	0.6643
		0.625	1	1	0.15	0.5056	0.3237	0.4615	0.5270	0.4212	1.559	1.479	1.155
					0.8133	0.6651	0.0	0.5373	1.411	0.4394	1.656	1.467	[illegible]
		1	1	-	0.05	0.4202	0.2900	0.4021	0.3614	0.3834	1.430	1.394	1.087
					0.15	0.4850	[illegible]	0.4573	0.5303	0.4320	1.571	1.521	1.166
					0.4	0.5492	0.3096	0.5008	0.8823	0.4608	1.654	1.576	1.157
					0.6	0.5808	0.2463	0.5164	1.150	0.4655	1.674	1.575	1.063
					0.8120	0.6083	0.0	0.5274	1.428	0.4653	1.682	1.561	0.7109
					1.1	0.6420	1.492	0.5447	1.659	[illegible]	1.702	1.569	0.8969
	1	1	1	-	0.15	0.5085	0.3381	0.4781	0.5563	0.4529	1.660	1.610	1.240
					0.4	0.5331	0.3051	0.4824	0.8715	0.4460	1.648	1.576	1.174
					0.6	0.5525	0.2459	0.4857	1.121	0.4404	1.638	1.549	1.071
					0.8290	0.5744	0.0	0.4894	1.403	0.4341	1.627	1.520	0.6798
0.2	0.5	0.3333	1	1	0.15	0.6229	0.3336	0.4808	0.4625	0.3458	1.451	1.198	1.064
					0.7905	1.033	0.0	0.6313	1.214	0.3112	1.562	1.023	0.8013
		0.625	1	1	0.15	0.5441	0.3235	0.4668	0.4713	0.3918	1.489	1.349	1.095
					0.7921	0.8240	0.0	0.5986	1.286	0.4105	[illegible]	1.319	0.7607
		1	1	-	0.05	0.4015	0.2657	0.3718	0.2977	0.3366	1.285	1.230	0.9808
					0.15	0.5091	0.3188	0.4604	0.4752	0.4114	1.507	1.419	1.110
					0.4	0.6243	0.3209	0.5370	0.8161	0.4550	1.640	1.499	1.139

Table 2, cont.

β	ω	t_s	σ	$\left(\frac{u_e}{u_\infty}\right)^2$	t_w	f"(0)	θ'(0)	g'(0)	I_1	I_2	$I_1(1)$	$I_1(2)$	$I_1(3)$
0.2	0.5	1	1	-	0.6	0.6818	0.2467	0.5655	1.069	0.4608	1.673	1.494	1.062
					0.7897	0.7268	0.0	0.5838	1.303	0.4589	1.686	1.474	0.8130
					1.1	0.7913	1.748	0.6060	1.659	0.4479	1.680	1.425	2.337
	0.7	0.3333	1	1	0.15	0.6428	0.3431	0.4908	0.4769	0.3568	1.495	1.240	1.098
					0.6	0.9193	0.2732	0.5747	1.011	0.3134	1.546	1.062	0.9341
					0.8123	1.024	0.0	0.5951	1.243	0.2787	1.535	0.9563	0.5252
		0.5	1	1	0.15	0.5828	0.3352	0.4798	0.4843	0.3892	1.526	1.350	1.122
					0.8099	0.8663	0.0	0.5727	1.275	0.3566	1.587	1.184	0.5789
		0.625	1	1	0.15	0.5581	0.3318	0.4752	0.4875	0.4021	1.539	1.397	1.133
					0.6	0.7377	0.2567	0.5471	1.047	0.4028	1.614	1.340	0.9888
					0.8085	0.8004	0.0	0.5627	1.290	0.3878	1.612	1.282	0.6020
		1	1	-	0.05	0.4445	0.2950	0.4103	0.3356	0.3766	1.421	1.358	1.076
					0.15	0.5202	0.3265	0.4678	0.4925	0.4216	1.560	1.470	1.150
					0.4	0.6068	0.3147	0.5155	0.8141	0.4426	1.633	1.495	1.136
					0.6	0.6548	0.2470	0.5338	1.059	0.4415	1.646	1.473	1.036
					0.8064	0.6979	0.0	0.5465	1.314	0.4350	1.653	1.439	0.6471
					1.1	0.7539	1.489	0.5607	1.657	0.4223	1.647	1.386	2.709
	1	0.3333	1	1	0.15	0.6768	0.3604	0.5112	0.4959	0.3793	1.565	1.316	1.160
					0.8297	1.001	0.0	0.5550	1.207	0.2769	1.458	0.9579	0.4980
		0.625	1	1	0.15	0.5840	0.3475	0.4935	0.5082	0.4214	1.615	1.475	1.199
					0.8257	0.7700	0.0	0.5215	1.260	0.3670	1.540	1.251	0.5734

Table 2, cont.

β	ω	t_s	σ	$\left(\frac{u_e}{u_\infty}\right)^2$	t_w	$f''(0)$	$\theta'(0)$	$g'(0)$	I_1	I_2	$I_1(1)$	$I_1(2)$	$I_1(3)$
0.2	1	1	1	-	0.15	0.5424	0.3414	0.4851	0.5143	0.4395	1.639	1.550	1.219
					0.4	0.5883	0.3085	0.4926	0.8065	0.4270	1.618	1.491	1.141
					0.8235	0.6633	0.0	0.5044	1.288	0.4057	1.586	1.397	0.6165
0.2857	0.5	0.3333	1	1	0.15	0.6898	0.3436	0.4965	0.4359	0.3175	1.434	1.101	1.044
					0.7933	1.209	0.0	0.6565	1.168	0.2198	1.553	0.7817	0.6157
		0.625	1	1	0.15	0.5843	0.3307	0.4787	0.4461	0.3792	1.481	1.296	1.081
					0.7887	0.9251	0.0	0.6191	1.210	0.3727	1.632	1.196	0.7071
		1	0	-	0.15	0.4283	0.3157	0.3643	0.3292	0.3342	1.360	1.302	1.144
					1.0	0.7627	0.4534	0.5179	1.309	0.3887	1.551	1.309	1.280
			0.5	-	0.15	0.4605	0.3101	0.3934	0.3743	0.3537	1.406	1.328	1.123
					0.9071	0.7683	0.0	0.5397	1.282	0.4019	1.595	1.335	0.5663
			1	-	0.05	0.4209	0.2710	0.3808	0.2779	0.3299	1.268	1.199	0.9696
					0.15	0.5371	0.3246	0.4704	0.4503	0.4053	1.502	1.385	1.100
					0.4	0.6701	0.3266	0.5505	0.7737	0.4428	1.631	1.446	1.120
					0.6	0.7401	0.2490	0.5810	1.013	0.4439	1.651	1.426	1.032
					0.7849	0.7952	0.0	0.6012	1.223	0.4374	1.667	1.393	0.7890
					1.1	0.8787	1.862	0.6294	1.548	0.4195	1.640	1.325	2.232
	0.7	0.3333	1	1	0.15	0.7114	0.3521	0.5050	0.4478	0.3278	1.473	1.139	1.075
					0.8093	1.197	0.0	0.6200	1.159	0.2053	1.496	0.7527	[illegible]
		0.5	1	1	0.15	0.6315	0.3421	0.4913	0.4561	0.3710	1.510	1.280	1.104
					0.8062	0.9897	0.0	0.5928	1.193	0.3089	1.556	1.034	0.5[illegible]90

Table 2, cont.

β	ω	c_s	σ	$\left(\frac{u_e}{u_\infty}\right)^2$	t_w	$f''(0)$	$\theta'(0)$	$g'(0)$	I_1	I_2	$I_1(1)$	$I_1(2)$	$I_1(3)$
0.2857	0.7	0.625	1	1	0.15	0.5984	0.3378	0.4853	0.4598	0.3883	1.526	1.339	1.117
					0.8049	0.9027	0.0	0.5806	1.209	0.3504	1.585	1.157	0.5540
		1	1	-	0.05	0.4636	0.2988	0.4171	0.3143	0.3707	1.411	1.329	1.068
					0.15	0.5475	0.3310	0.4758	0.4657	0.4143	1.553	1.433	1.138
					0.4	0.6512	0.3194	0.5259	0.7744	0.4298	1.624	1.439	1.108
					0.6	0.7118	0.2499	0.5459	1.007	0.4241	1.635	1.402	0.9885
					0.8022	0.7664	0.0	0.5603	1.236	0.4133	1.635	1.356	0.6055
					1.1	0.8401	1.565	0.5771	1.557	0.3950	1.621	1.286	2.709
	1	1	1	-	0.15	0.5684	0.3438	0.4901	0.4846	0.4299	1.625	1.508	1.204
					0.4	0.6308	0.3108	0.4999	0.7611	0.4134	1.598	1.432	1.117
					0.6	0.6793	0.2473	0.5072	0.9773	0.4001	1.578	1.374	0.9918
					0.8194	0.7310	0.0	0.5148	1.210	0.3854	1.558	1.314	0.5758
0.4	0.5	0.3333	1	0	0.15	0.6910	0.3363	0.4393	0.3396	0.2834	1.311	1.024	1.030
					0.8938	1.434	0.0	0.6268	1.081	0.1360	1.410	0.5590	0.4240
				0.5	0.15	0.7224	0.3403	0.4643	0.3656	0.2837	1.346	1.012	1.022
					0.8498	1.431	0.0	0.6476	1.087	0.1264	1.452	0.5407	0.4055
				1	0.15	0.7703	0.3550	0.5146	0.4069	0.2845	1.418	0.9944	1.023
					0.7898	1.412	0.0	0.6872	1.084	0.1311	1.523	0.5552	0.5541
		0.625	1	0	0.15	0.5342	0.3219	0.3972	0.3279	0.3244	1.342	1.204	1.092
					0.9407	1.044	0.0	0.5669	1.156	0.3039	1.484	1.009	0.5241
				0.5	0.15	0.5674	0.3221	0.4255	0.3616	0.3359	1.383	1.213	1.077
					0.8738	1.044	0.0	0.5893	1.149	0.3085	1.530	1.019	0.4872

Table 2, cont.

β	ω	t_s	σ	$\left(\frac{u_e}{u_\infty}\right)^2$	t_w	$f''(0)$	$\theta'(0)$	$g'(0)$	I_1	I_2	$I_1(1)$	$I_1(2)$	$I_1(3)$
0.4	0.5	0.625	1	1	0.15	0.6332	0.3393	0.4928	0.4182	0.3648	1.473	1.236	1.067
					0.7829	1.045	0.0	0.6439	1.117	0.3334	1.599	1.070	0.6992
		1	0	-	0.15	0.4540	0.3192	0.3686	0.3094	0.3303	1.347	1.273	1.133
					1.0	0.8544	0.4632	0.5300	1.219	0.3667	1.521	1.219	1.257
			0.5	-	0.15	0.4887	0.3141	0.3992	0.3511	0.3483	1.394	1.296	1.111
					0.9048	0.8564	0.0	0.5533	1.194	0.3786	1.570	1.244	0.5232
			1	-	0.05	0.4416	0.2757	0.3870	0.2694	0.3321	1.289	1.188	0.9672
					0.15	0.5714	0.3318	0.4824	0.4229	0.3987	1.499	1.349	1.090
					0.4	0.7260	0.3334	0.5667	0.7270	0.4291	1.624	1.386	1.099
					0.6	0.8112	0.2515	0.5995	0.9516	0.4246	1.651	1.351	0.9983
					0.7815	0.8782	0.0	0.6203	1.149	0.4135	1.659	1.306	0.7155
					1.1	0.9837	1.924	0.6489	1.474	0.3884	1.644	1.218	2.562
	0.7	0.3333	1	0	0.15	0.7382	0.3634	0.4742	0.3620	0.3039	1.385	1.071	1.081
					0.8964	1.428	0.0	0.6161	1.072	0.1408	1.390	0.5692	0.4242
				0.5	0.15	0.7613	0.3602	0.4911	0.3841	0.3005	1.406	1.055	1.065
					0.8545	1.418	0.0	0.6275	1.070	0.1346	1.414	0.5585	0.4054
				1	0.15	0.7938	0.3622	0.5212	0.4162	0.2945	1.450	1.029	1.052
					0.8052	1.403	0.0	0.6471	1.068	0.1234	1.458	0.5395	0.4289
		0.625	1	0	0.15	0.5807	0.3550	0.4369	0.3527	0.3550	1.429	1.274	1.152
					0.9414	1.040	0.0	0.5619	1.152	0.3027	1.475	1.007	0.5242
				0.5	0.15	0.6049	0.3457	0.4564	0.3834	0.3594	1.456	1.272	1.126
					0.8763	1.032	0.0	0.5743	1.136	0.3044	1.502	1.013	0.4875

Table 2, cont.

β	ω	t_s	σ	$\left(\frac{u_e}{u_\infty}\right)^2$	t_w	$f''(0)$	$\theta'(0)$	$g'(0)$	I_1	I_2	$I_1(1)$	$I_1(2)$	$I_1(3)$
0.4	0.7	0.625	1	1	0.15	0.6473	0.3448	0.4971	0.4292	0.3726	1.513	1.275	1.100
					0.8005	1.025	0.0	0.6007	1.122	0.3077	1.559	1.021	0.5053
		1	0	-	0.15	0.5027	0.3597	0.4129	0.3347	0.3687	1.443	1.358	1.204
					1.0	0.8544	0.4632	0.5300	1.219	0.3667	1.521	1.219	1.257
			0.5	-	0.15	0.5281	0.3422	0.4344	0.3749	0.3781	1.475	1.366	1.166
					0.4	0.6583	0.3671	0.4909	0.6569	0.3955	1.544	1.362	1.176
					0.6	0.7386	0.3500	0.5159	0.8701	0.3906	1.554	1.318	1.119
					0.9058	0.8470	0.0	0.5430	1.185	0.3728	1.551	1.235	0.5240
			1	-	0.05	0.4864	0.3034	0.4246	0.2943	0.3654	1.404	1.300	1.058
					0.15	0.5807	0.3363	0.4853	0.4358	0.4063	1.546	1.393	1.126
					0.4	0.7053	0.3239	0.5389	0.7220	0.4154	1.608	1.377	1.092
					0.6	0.7811	0.2500	0.5609	0.9380	0.4050	1.614	1.326	0.9700
					0.7974	0.8490	0.5684	0.5763	1.151	0.3887	1.617	1.265	0.5624
					1.1	0.9451	1.636	0.5953	1.460	0.3616	1.603	1.172	2.887
	1	0.3333	1	1	0.15	0.8319	0.3759	0.5366	0.4289	0.3173	1.500	1.098	1.103
					0.8231	1.383	0.0	0.5993	1.033	0.1439	1.363	0.5815	0.3952
		0.625	1	1	0.15	0.6727	0.3563	0.5094	0.4438	0.3887	1.571	1.342	1.158
					0.8179	0.9951	0.0	0.5520	1.091	0.2967	1.467	1.006	0.4799
		1	0	-	0.15	0.5901	0.4327	0.4926	0.3806	0.4380	1.619	1.514	1.333
					1.0	0.8544	0.4632	0.5300	1.219	0.3667	1.521	1.219	1.257
			0.5	-	0.15	0.5948	0.3901	0.4942	0.4156	0.4286	1.614	1.487	1.261
					0.9074	0.8337	0.0	0.5283	1.173	0.3646	1.525	1.221	0.5248

Table 2, cont.

β	ω	t_s	σ	$\left(\frac{u_e}{u_\infty}\right)^2$	t_w	$f'(0)$	$\theta'(0)$	$g'(0)$	I_1	I_2	$I_1(1)$	$I_1(2)$	$I_1(3)$
0.4	1	1	1	-	0.15	0.5998	0.3465	0.4959	0.4515	0.4192	1.608	1.460	1.187
					0.4	0.6824	0.3134	0.5082	0.7109	0.3980	1.575	1.366	1.092
					0.6	0.7461	0.2474	0.5173	0.9129	0.3809	1.552	1.295	0.9564
					0.8147	0.8125	0.0	0.5264	1.125	0.3624	1.529	1.224	0.5334
0.5	0.5	0.3333	1	0.5	0.15	0.7824	0.3469	0.4739	0.3450	0.2644	1.328	0.9453	1.006
					0.8488	1.599	0.0	0.6662	1.022	0.0638	1.424	0.3868	0.3726
		1	0	-	0.05	0.3518	0.2520	0.2923	0.1704	0.2666	1.153	1.103	0.9820
					0.15	0.4747	0.3218	0.3720	0.2946	0.3274	1.336	1.251	1.125
					0.4	0.6472	0.3940	0.4533	0.5628	0.3658	1.455	1.290	1.212
					0.6	0.7511	0.4266	0.4899	0.7652	0.3685	1.484	1.258	1.232
					1.0	0.9277	0.4705	0.5390	1.155	0.3503	1.500	1.155	1.240
					1.1	0.9681	0.4791	0.5486	1.250	0.3431	1.500	1.126	1.240
			0.25	-	0.05	0.3653	0.2517	0.3038	0.1840	0.2733	1.171	1.112	0.9772
					0.15	0.4909	0.3187	0.3857	0.3127	0.3345	1.358	1.259	1.114
					0.9523	0.9260	0.0	0.5494	1.142	0.3549	1.522	1.165	0.4897
			0.5	-	0.05	0.3827	0.2532	0.3189	0.2001	0.2826	1.194	1.124	0.9728
					0.15	0.5114	0.3171	0.4037	0.3339	0.3444	1.385	1.271	1.103
					0.4	0.6867	0.3637	0.4878	0.6147	0.3794	1.507	1.303	1.147
					0.6	0.7908	0.3539	0.5247	0.8245	0.3786	1.537	1.264	1.098
					0.903	0.9266	0.0	0.5635	1.131	0.3612	1.552	1.179	0.4935
					1.1	1.007	1.024	0.5830	0.1325	0.3446	1.553	1.117	2.080
			0.6	-	0.05	0.3913	0.2546	0.3265	0.2077	0.2874	1.206	1.130	0.9712
					0.15	0.5215	0.3173	0.4128	0.3438	0.3496	1.399	1.277	1.098
					0.8826	0.9280	0.0	0.5708	1.127	0.3645	1.566	1.185	0.4953

Table 2, cont.

β	ω	t_s	σ	$\left(\frac{u_e}{u_\infty}\right)^2$	t_w	$f''(0)$	$\theta'(0)$	$g'(0)$	I_1	I_2	$I_1(1)$	$I_1(2)$	$I_1(3)$
0.5	0.5	1	0.8	-	0.05	0.4138	0.2603	0.3471	0.2261	0.3007	1.236	1.146	0.9680
					0.1	0.4928	0.2989	0.4028	0.3017	0.3419	1.366	1.249	1.052
					0.15	0.5475	0.3204	0.4371	0.3675	0.3638	1.435	1.294	1.090
					0.8393	0.9335	0.0	0.5906	1.121	0.3731	1.605	1.201	0.4989
			1	-	0.05	0.4614	0.2815	0.3970	0.2496	0.3233	1.258	1.155	0.9530
					0.15	0.6009	0.3387	0.4957	1.386	0.3871	1.460	1.306	1.074
					0.4	0.7723	0.3385	0.5831	0.6808	0.4149	1.586	1.334	1.089
					0.6	0.8688	0.2521	0.6156	0.8997	0.4094	1.629	1.294	0.9863
					0.7762	0.9439	0.0	0.6374	1.080	0.3958	1.634	1.242	0.7265
					1.1	1.069	2.001	0.6666	1.404	0.3640	1.632	1.138	2.655
	0.7	1	0	-	0.05	0.4252	0.3109	0.3578	0.2008	0.3248	1.307	1.244	1.098
					0.15	0.5243	0.3625	0.4164	0.3182	0.3652	1.433	1.334	1.196
					0.4	0.6713	0.4142	0.4750	0.5779	0.3823	1.499	1.324	1.244
					0.6	0.7643	0.4378	0.5018	0.7751	0.3768	1.508	1.275	1.249
					1.0	0.9277	0.4705	0.5390	1.155	0.3503	1.500	1.155	1.240
					1.1	0.9657	0.4770	0.5464	1.247	0.3420	1.496	1.124	1.237
			0.5	-	0.05	0.4500	0.3013	0.3784	0.2322	0.3339	1.339	1.253	1.075
					0.15	0.5513	0.3452	0.4388	0.3560	0.3733	1.465	1.340	1.157
					0.4	0.6986	0.3710	0.4975	0.6238	0.3867	1.530	1.321	1.162
					0.6	0.7913	0.3535	0.5238	0.8258	0.3784	1.538	1.266	1.101
					0.9040	0.9169	0.0	0.5526	1.122	0.3557	1.532	1.170	0.4941
					1.1	0.9920	0.9870	0.5674	1.308	0.3379	1.524	1.105	2.030
			1	-	0.05	0.5048	0.3071	0.4305	0.2794	0.3616	1.400	1.278	1.051
					0.15	0.6073	0.3405	0.4928	0.4134	0.4004	1.540	1.363	1.118
					0.4	0.7489	0.3277	0.5488	0.6862	0.4047	1.599	1.332	1.076

Table 2. cont.

β	ω	t_s	σ	$\left(\frac{u_e}{u_\infty}\right)^2$	t_w	$f''(0)$	$\theta'(0)$	$g'(0)$	I_1	I_2	$I_1(1)$	$I_1(2)$	$I_1(3)$
0.5	0.7	1	1	-	0.6	0.8370	0.2518	0.5719	0.8938	0.3900	1.607	1.268	0.9366
					0.7934	0.9146	0.0	0.5886	1.087	0.3711	1.603	1.200	0.5470
					1.1	1.029	1.700	0.6095	1.386	0.3372	1.588	1.090	2.953
	1	1	0	-	0.15	0.6136	0.4358	0.4964	0.3609	0.4331	1.609	1.487	1.325
					0.4	0.7105	0.4471	0.5103	0.6026	0.4092	1.572	1.380	1.296
					0.6	0.7850	0.4555	0.5205	0.7906	0.3897	1.545	1.301	1.276
					1.0	0.9277	0.5390	0.5390	1.155	0.3503	1.500	1.155	1.240
			1	-	0.15	0.6249	0.3485	0.5003	0.4272	0.4113	1.596	1.425	1.174
					0.4	0.7237	0.3152	0.5145	0.6743	0.3866	1.559	1.318	1.073
					0.6	0.7997	0.2474	0.5248	0.8662	0.3667	1.533	1.238	0.9303
					0.8111	0.8773	0.0	0.5349	1.064	0.3454	1.508	1.159	0.5038
					1.1	0.9800	1.409	0.5478	1.328	0.3161	1.478	1.058	2.702
0.75	0.5	1	0.5	-	0.05	0.4131	0.2576	0.3251	0.1796	0.2784	1.182	1.092	0.9608
					0.15	0.5618	0.3234	0.4129	0.3000	0.3367	1.368	1.223	1.086
					0.4	0.7768	0.3721	0.5020	0.5518	0.3630	1.483	1.224	1.121
					0.6	0.9092	0.3616	0.5419	0.7391	0.3549	1.508	1.165	1.064
					0.8994	1.083	0.0	0.5840	1.008	0.3264	1.518	1.052	0.4382
					1.1	1.189	1.085	0.6061	1.183	0.3017	1.517	0.9717	2.118
	0.7	1	0	-	0.05	0.4557	0.3150	0.3629	0.1800	0.3207	1.292	1.211	1.085
					0.15	0.5726	0.3683	0.4237	0.2857	0.3582	1.412	1.288	1.179
					0.4	0.7568	0.4232	0.4862	0.5183	0.3682	1.470	1.251	1.221
					0.6	0.8770	0.4489	0.5155	0.6939	0.3565	1.474	1.183	1.223
					1.0	1.090	0.4849	0.5568	1.031	0.3174	1.460	1.031	1.209
					1.1	1.140	0.4922	0.5652	1.113	0.3059	1.454	0.9922	1.204

Table 2, cont.

β	ω	t_s	σ	$\left(\frac{u_e}{u_\infty}\right)^2$	t_w	$f''(0)$	$\theta'(0)$	$g'(0)$	I_1	I_2	$I_1(1)$	$I_1(2)$	$I_1(3)$
0.75	0.7	1	0.5	-	0.05	0.4833	0.3059	0.3850	0.2072	0.3279	1.325	1.215	1.061
					0.15	0.6029	0.3512	0.4478	0.3187	0.3690	1.446	1.287	1.139
					0.4	0.7885	0.3787	0.5108	0.5591	0.3692	1.503	1.240	1.135
					0.6	0.9089	0.3605	0.5400	0.7396	0.3542	1.507	1.166	1.065
					0.9004	1.072	0.0	0.5719	1.0000	0.3215	1.496	1.044	0.4385
					1.1	1.173	1.045	0.5890	1.169	0.2963	1.485	0.9624	2.062
1	0.5	0.3333	1	0	0.05	0.6684	0.2862	0.3695	0.1550	0.2033	1.075	0.7781	0.8644
					0.15	0.9742	0.3626	0.4740	0.2546	0.2097	1.228	0.7633	0.9629
					0.8980	2.298	0.0	0.6980	0.8082	-0.1293	1.287	-0.0618	0.2881
		0.625	1	0	0.05	0.4913	0.2665	0.3280	0.1496	0.2520	1.115	0.9741	0.9235
					0.15	0.6981	0.3394	0.4198	0.2520	0.2948	0.1280	1.052	1.042
					0.9429	1.580	0.0	0.6190	0.8788	0.1736	1.378	0.6162	0.4018
		1	0	-	0.05	0.4025	0.2584	0.3004	0.1408	0.2617	1.127	1.053	0.9605
					0.15	0.5612	0.3317	0.3844	0.2437	0.3176	1.300	1.175	1.096
					0.4	0.8060	0.4097	0.4730	0.4631	0.3430	1.404	1.167	1.172
					0.6	0.9615	0.4460	0.5140	0.6272	0.3344	1.426	1.102	1.186
					1.0	1.233	0.4959	0.5705	0.9402	0.2923	1.430	0.9402	1.186
					1.1	1.295	0.5058	0.5817	1.016	0.2791	1.429	0.8978	1.184
			0.25	-	0.05	0.4187	0.2588	0.3132	0.1514	0.2673	1.146	1.059	0.9556
					0.15	0.5808	0.3291	0.3998	0.2578	0.3230	1.323	1.180	1.084
					0.9490	1.224	0.0	0.5829	0.9289	0.2955	1.458	0.9490	0.3943
			0.5	-	0.05	0.4395	0.2612	0.3302	0.1641	0.2755	1.172	1.068	0.9516
					0.15	0.6057	0.3285	0.4204	0.2746	0.3311	1.355	1.187	1.074
					0.4	0.8558	0.3787	0.5132	0.5051	0.3507	1.465	1.166	1.102

Table 2, cont.

β	ω	t_s	σ	$\left(\frac{u_e}{u_\infty}\right)^2$	t_w	$f''(0)$	$\theta'(0)$	$g'(0)$	I_1	I_2	$I_1(1)$	$I_1(2)$	$I_1(3)$
1	0.5	1	0.5	-	0.6	1.013	0.3675	0.5554	0.6760	0.3369	1.488	1.092	1.039
					0.8965	1.219	0.0	0.6000	0.9187	0.3000	1.495	0.9601	0.3992
					1.1	1.349	1.135	0.6241	1.080	0.2689	1.492	0.8652	2.149
			0.6	-	0.05	0.4499	0.2632	0.3390	0.1700	0.2799	1.185	1.073	0.9504
					0.1	0.5467	0.3047	0.3954	0.2298	0.3169	1.307	1.159	1.033
					0.15	0.6181	0.3292	0.4310	0.2825	0.3356	1.371	1.192	1.070
					0.8748	1.218	0.0	0.6091	0.9152	0.3025	1.513	0.9655	0.4013
			0.8	-	0.05	0.4774	0.2708	0.3632	0.1848	0.2926	1.221	1.086	0.9490
					0.1	0.5774	0.3115	0.4227	0.2472	0.3302	1.348	1.173	1.029
					0.15	0.6501	0.3344	0.4599	0.3016	0.3486	1.415	1.205	1.063
					0.8290	1.218	0.0	0.6344	0.9094	0.3090	1.565	0.9788	0.4061
			1	-	0.05	0.5381	0.3014	0.4277	0.2068	0.3133	1.244	1.087	0.9266
					0.15	0.7166	0.3623	0.5342	0.3297	0.3745	1.473	1.221	1.049
					0.4	0.9653	0.3634	0.6465	0.5532	0.3694	1.488	1.163	1.016
					0.6	1.110	0.2598	0.6760	0.7445	0.3577	1.603	1.104	0.8990
					0.7640	1.220	0.0	0.7006	0.8838	0.3329	1.606	1.027	0.6067
					1.1	1.423	2.402	0.7483	1.136	0.2860	1.517	0.8877	2.484
	0.7	0.3333	1	0	0.05	0.7685	0.3367	0.4338	0.1784	0.2328	1.208	0.8477	0.9573
					0.15	1.034	0.3912	0.5111	0.2701	0.2224	1.299	0.7915	1.012
					0.9006	2.295	0.0	0.6871	0.8031	-0.1164	1.265	-0.0368	0.2890
		0.625	1	0	0.05	0.5761	0.3206	0.3929	0.1735	0.2985	1.260	1.084	1.028
					0.15	0.7516	0.3736	0.4609	0.2692	0.3208	1.366	1.109	1.102
					0.9435	1.577	0.0	0.6140	0.8765	0.1743	1.368	0.6178	0.4019
		1	0	-	0.05	0.4821	0.3183	0.3670	0.1643	0.3177	1.280	1.186	1.076
					0.15	0.6146	0.3729	0.4295	0.2614	0.3532	1.396	1.253	1.167

Table 2, cont.

β	ω	t_s	σ	$\left(\frac{u_e}{u_\infty}\right)^2$	t_w	$f''(0)$	$\theta'(0)$	$g'(0)$	I_1	I_2	$I_1(1)$	$I_1(2)$	$I_1(3)$
1	0.7	1	0	-	0.4	0.8317	0.[illegible]302	0.4950	0.4741	0.3576	1.449	1.197	1.205
					0.6	0.9756	0.4574	0.5262	0.6343	0.3413	1.450	1.116	1.203
					1.0	1.233	0.4959	0.5705	0.9402	0.2923	1.430	0.9402	1.186
					1.1	1.293	0.5037	0.5795	1.014	0.2783	1.424	0.8964	1.181
			0.5	-	0.05	0.5120	0.3095	0.3902	0.1884	0.3235	1.314	1.187	1.051
					0.15	0.6478	0.3560	0.4550	0.2908	0.3570	1.432	1.248	1.126
					0.4	1.505	1.024	1.366	0.2288	0.1155	0.4640	0.4537	0.3749
					0.6	1.012	0.3658	0.5526	0.6759	0.3359	1.484	1.092	1.039
					0.8975	1.207	0.0	0.5869	0.9113	0.2955	1.470	0.9523	0.3995
					1.1	1.331	1.091	0.6057	1.067	0.2644	1.457	0.8578	2.089
			0.9	-	0.15	0.6940	0.3506	0.4955	0.3239	0.3702	1.491	1.254	1.095
					0.4	0.9117	0.3494	0.5617	0.5505	0.3612	1.544	1.173	1.037
					0.6	1.055	0.2822	0.5919	0.7195	0.3351	1.543	1.075	0.8892
					0.8093	1.192	0.0	0.6160	0.8902	0.3019	1.534	0.9683	0.4096
			1	-	0.05	0.5809	0.3222	0.4548	0.2275	0.3488	1.387	1.202	1.026
					0.15	0.7188	0.3576	0.5230	0.3372	0.3810	1.528	1.262	1.088
					0.4	0.9317	0.3426	0.5891	0.5616	0.3668	1.562	1.175	1.023
					0.6	1.071	0.2551	0.6162	0.7336	0.3379	1.579	1.075	0.8527
					0.7779	1.185	0.0	0.6372	0.8753	0.3108	1.549	0.9880	0.5073
					1.1	1.380	1.970	0.6650	1.130	0.2547	1.527	0.8266	3.035
	1	0.1539	1	0	0.15	1.764	0.4779	0.6465	0.2809	0.0612	1.272	0.1107	0.9563
					0.8838	3.839	0.0	0.7797	0.7071	-0.7567	1.077	-1.207	0.1603
		0.3333	1	0	0.15	1.132	0.4391	0.5730	0.2962	0.2436	1.420	0.8397	1.094
					0.9045	2.290	0.0	0.6715	0.7960	-0.0978	1.236	-0.0008	0.2903

Table 2, cont.

β	ω	t_s	σ	$\left(\frac{u_e}{u_\infty}\right)^2$	t_w	$f''(0)$	$\theta'(0)$	$g'(0)$	I_1	I_2	$I_1(1)$	$I_1(2)$	$I_1(3)$
1	1	0.625	1	0	0.15	0.8444	0.4329	0.5321	0.2992	0.3657	1.516	1.210	1.207
					0.9444	1.572	0.0	0.6067	0.8733	0.1753	1.354	0.6204	0.4024
		1	0	-	0.15	0.7104	0.4473	0.5107	0.2934	0.4171	1.572	1.395	1.296
					0.4	0.8735	0.4637	0.5308	0.4921	0.3814	1.521	1.246	1.257
					1.0	1.233	0.4[illegible]59	0.5705	0.9402	0.2923	1.430	0.9402	1.186
1.4	0.7	1	0.6	-	0.15	0.7200	0.3596	0.4719	0.2640	0.3505	1.426	1.200	1.101
					0.4	0.9874	0.3830	0.5427	0.4622	0.3403	1.472	1.105	1.071
					0.6	1.167	0.3516	0.5765	0.6100	0.3125	1.469	0.9983	0.9707
					0.8721	1.389	0.0	0.6109	0.8028	0.2653	1.453	0.8486	0.3577
					1.1	1.564	1.295	0.6343	0.9587	0.2214	1.437	0.7246	2.341
1.5	0.5	1	0	-	0.05	0.4427	0.2628	0.3060	0.1225	0.25[illegible]	1.110	1.022	0.9465
					0.15	0.6304	0.3383	0.3928	0.2123	0.3119	1.277	1.128	1.078
					0.4	0.9337	0.4200	0.4860	0.4030	0.3291	1.374	1.092	1.148
					0.6	1.131	0.4585	0.5298	0.5448	0.3134	1.391	1.008	1.159
					1.0	1.477	0.5119	0.5906	0.8141	0.2562	1.390	0.8141	1.155
			0.25	-	0.05	0.4608	0.2636	0.3197	0.1312	0.2640	1.130	1.026	0.9418
					0.15	0.6526	0.3361	0.4095	0.2242	0.3163	1.302	1.130	1.066
					0.9469	1.464	0.0	0.6045	0.8034	0.2585	1.421	0.8216	0.3415
			0.5	-	0.05	0.4842	0.2667	0.3381	0.1419	0.2715	1.158	1.033	0.9384
					0.15	0.6810	0.3362	0.4319	0.2383	0.3234	1.336	1.136	1.056
					0.4	0.9917	0.3888	0.5304	0.4389	0.3332	1.440	1.084	1.075
					0.6	1.192	0.3763	0.5760	0.5871	0.3111	1.459	0.9885	1.004
					0.8922	1.453	0.0	0.6241	0.7937	0.2620	1.463	0.8311	0.3466

Table 2, cont.

β	ω	t_s	σ	$\left(\frac{u_e}{u_\infty}\right)^2$	t_w	$f''(0)$	$\theta'(0)$	$g'(0)$	I_1	I_2	$I_1(1)$	$I_1(2)$	$I_1(3)$
1.5	0.7	1	0	-	0.05	0.5266	0.3233	0.3733	0.1419	0.3138	1.262	1.150	1.061
					0.6	1.145	0.4700	0.5420	0.5504	0.3196	1.415	1.021	1.176
					1.0	1.477	0.5119	0.5906	0.8140	0.2561	1.390	0.8140	1.155
1.8	0.5	1	0.6	-	0.1	0.6421	0.3155	0.4115	0.1856	0.3083	1.286	1.095	1.010
					0.15	0.7354	0.3414	0.4496	0.2285	0.3241	1.347	1.116	1.044
					0.4	1.080	0.3877	0.5524	0.4182	0.3273	1.451	1.047	1.048
					0.6	1.303	0.3616	0.6000	0.5578	0.2993	1.469	0.9388	0.9526
					1.1	1.788	1.425	0.6766	0.8857	0.1938	1.467	0.6373	2.484
2	0.5	1	0	-	0.05	0.4766	0.2661	0.3103	0.1097	0.2573	1.097	0.9992	0.9364
					0.15	0.6893	0.3432	0.3991	0.1906	0.3082	1.261	1.096	1.065
					0.4	1.043	0.4276	0.4956	0.3617	0.3196	1.352	1.040	1.131
					0.6	1.276	0.4677	0.5414	0.4884	0.2988	1.367	0.9445	1.140
					1.0	1.687	0.5235	0.6052	0.7282	0.2308	1.363	0.7282	1.134
			0.25	-	0.05	0.4962	0.2672	0.3246	0.1172	0.2619	1.119	1.003	0.9319
					0.15	0.7137	0.3414	0.4167	0.2009	0.3119	1.287	1.096	1.053
					0.9452	1.669	0.0	0.6204	0.7180	0.2326	1.396	0.7348	0.3063
	0.7	1	0	-	0.05	0.5640	0.3270	0.3781	0.1264	0.3112	1.249	1.125	1.051
					0.15	0.7473	0.3851	0.4449	0.2028	0.3417	1.357	1.168	1.136
					0.4	1.071	0.4484	0.5179	0.3690	0.3326	1.396	1.067	1.164
					0.6	1.291	0.4793	0.5537	0.4930	0.3046	1.391	0.9561	1.158
					1.0	1.687	0.5235	0.6052	0.7282	0.2308	1.363	0.7282	1.134

Table 2, cont.

β	ω	t_s	σ	$\left(\frac{u_e}{u_\infty}\right)^2$	t_w	$f''(0)$	$\theta'(0)$	$g'(0)$	I_1	I_2	$I_1(1)$	$I_1(2)$	$I_1(3)$
3	0.5	1	0	-	0.15	0.7883	0.3503	0.4082	0.1616	0.3036	1.238	1.051	1.047
					0.4	1.228	0.4383	0.5093	0.3072	0.3074	1.323	0.9723	1.109*
					0.6	1.522	0.4805	0.5578	0.4142	0.2796	1.335	0.8603	1.115*
					1.0	2.044	0.5397	0.6258	0.6158	0.1967	1.328	0.6158	1.107
	0.7	1	0	-	0.05	0.6255	0.3324	0.3849	0.1057	0.3083	0.1231	1.091	1.037
					0.15	0.8493	0.3926	0.4545	0.1711	0.3360	1.334	1.121	1.118
					0.4	1.257	0.4593	0.5319	0.3125	0.3194	1.367	0.9973	1.141
					0.6	1.537	0.492[illegible]	0.5702	0.4176	0.2847	1.359	0.8708	1.133
					1.0	2.044	0.5[illegible]	0.6258	0.6158	0.1967	1.328	0.6158	1.107

*Convergence to 10^{-4}.

Table 3

SIMILAR SOLUTIONS FOR Pr = 0.7, $f_w \neq 0$, $t_s = 1.0$

This table is summarized on pages 31--33.

β	ω	f_w	σ_1	t_w	$f''(0)$	$\theta'(0)$	$g'(0)$	I_1	I_2	$I_1(1)$	$I_1(2)$	$I_1(3)$
0	0.5	0	1	0.15	0.4328	0.3026	0.4328	0.5519	0.4303	1.523	1.523	1.142
				0.8009	0.5343	0.0	0.5343	1.546	0.5239	1.742	1.742	0.9846
		-0.2	1	0.15	0.2920	0.2093	0.2920	0.6331	0.4888	1.836	1.836	1.415
				0.7755	0.3897	0.0	0.3897	1.697	0.5774	2.004	2.004	1.368
		-0.4	1	0.15	0.1663	0.1262	0.1663	0.7514	0.5620	2.318	2.318	1.843
				0.7455	0.2563	0.0	0.2563	1.898	0.6411	2.366	2.366	1.840
		-0.6	1	0.15	0.0629	0.0556	0.0629	0.9530	0.6548	3.224	3.224	2.672
				0.4	0.1135	0.0720	0.1135	1.517	0.6980	2.981	2.981	2.440
				0.6	0.1315	0.0435	0.1315	1.956	0.7123	2.934	2.934	2.444
	0.7	0	0	0.15	0.3920	0.3428	0.3920	0.4414	0.3921	1.508	1.508	1.255
				1.0	0.4696	0.4139	0.4696	1.686	0.4696	1.686	1.686	1.385
			0.5	0.15	0.4089	0.3250	0.4089	0.4970	0.4089	1.538	1.537	1.224
				0.9162	0.4802	0.0	0.4802	1.636	0.4802	1.700	1.700	0.7691
			1	0.15	0.4451	0.3136	0.4451	0.5766	0.4448	1.586	1.586	1.188
				0.8187	0.5034	0.0	0.5034	1.580	0.5029	1.725	1.725	0.8037
		-0.2	0	0.15	0.2559	0.2422	0.2559	0.5611	0.4559	1.869	1.869	1.539
				1.0	0.3305	0.3108	0.3305	1.999	0.3305	1.999	1.999	1.623
			0.5	0.15	0.2718	0.2281	0.2718	0.6022	0.4718	1.884	1.884	1.508
				0.9028	0.3400	0.0	0.3400	1.887	0.5400	2.004	2.004	1.210

Table 3, cont.

β	ω	f_w	σ_1	t_w	$f''(0)$	$\theta'(0)$	$g'(0)$	I_1	I_2	$I_1(1)$	$I_1(2)$	$I_1(3)$
0	0.7	-0.2	1	0.15	0.3050	0.2205	0.3050	0.6625	0.5048	1.901	1.901	1.458
				0.7925	0.3612	0.0	0.3612	1.757	0.5605	2.011	2.011	1.228
		-0.4	0	0.15	0.1371	0.1481	0.1371	0.7445	0.5371	2.443	2.443	1.998
				1.0	0.2049	0.2126	0.2049	2.447	0.6049	2.447	2.447	1.968
			0.5	0.15	0.1509	0.1398	0.1509	0.7580	0.5509	2.422	2.422	1.957
				0.8868	0.2128	0.0	0.2128	2.239	0.6128	2.438	2.438	1.763
			1	0.15	0.1795	0.1369	0.1795	0.7824	0.5792	2.373	2.373	1.872
				0.7616	0.2311	0.0	0.2311	1.996	0.6301	2.415	2.415	1.754
		-0.6	0	0.15	0.0433	0.0620	0.0433	1.081	0.6433	3.613	3.613	2.979
				0.4	0.0728	0.0941	0.0728	1.705	0.6728	3.321	3.321	[illegible]
				0.6	0.0843	0.1063	0.0843	2.200	0.6843	3.249	3.249	2.622
				1.0	0.0975	0.1201	0.0975	3.181	0.6975	3.181	3.181	2.552
			0.5	0.15	0.0533	0.0615	0.0533	1.032	0.6533	3.460	3.460	2.856
				0.8682	0.1033	0.0	0.1033	2.805	0.7034	3.143	3.143	2.563
			1	0.15	0.0745	0.0651	0.0745	0.9795	0.6737	3.214	3.214	2.629
				0.4	0.1041	0.0693	0.1041	1.591	0.7026	3.091	3.091	2.501
				0.6	0.1138	0.0432	0.1138	2.068	0.7119	3.063	3.063	2.486
				0.7248	0.1177	0.0	0.1177	2.364	0.7157	3.052	3.052	2.502
0.2	0.5	0	1	0.15	0.5091	0.3188	0.4604	0.4752	0.4114	1.507	1.419	1.110
				0.7897	0.7268	0.0	0.5838	1.303	0.4589	1.686	1.474	0.8130
		-0.2	1	0.15	0.3733	0.2286	0.3250	0.5289	0.4638	1.779	1.670	1.343
				0.7625	0.5874	0.0	0.4463	1.390	0.5021	1.901	1.653	1.107

Table 3, cont.

β	ω	f_w	σ_1	t_w	$f''(0)$	$\theta'(0)$	$g'(0)$	I_1	I_2	$I_1(1)$	$I_1(2)$	$I_1(3)$
0.2	0.5	-0.4	1	0.15	0.2525	0.1494	0.2056	0.5983	0.5277	2.154	2.017	1.670
				0.7282	0.4579	0.0	0.3216	1.476	0.5516	2.158	1.875	1.466
		-0.6	1	0.15	0.1517	0.0839	0.1080	0.6917	0.6061	2.701	2.526	2.158
				0.4	0.2611	0.0975	0.1760	1.111	0.6180	2.541	2.302	1.984
				0.6	0.3192	0.0498	0.2039	1.427	0.6110	2.485	2.193	1.916
				0.6882	0.3406	0.0	0.2114	1.586	0.6115	2.498	2.165	1.855
	0.7	0	1	0.15	0.5202	0.3265	0.4678	0.4925	0.4216	1.560	1.470	1.150
				0.8064	0.6979	0.0	0.5465	1.314	0.4350	1.653	1.439	0.6471
		-0.2	1	0.15	0.3846	0.2361	0.3329	0.5479	0.4749	1.836	1.724	1.383
				0.7765	0.5614	0.0	0.4122	1.408	0.4797	1.880	1.628	0.9855
		-0.4	1	0.15	0.2633	0.1562	0.2135	0.6179	0.5395	2.212	2.070	1.708
				0.7400	0.4352	0.0	0.2910	1.511	0.5328	2.164	1.866	1.367
		-0.6	1	0.15	0.1610	0.0896	0.1150	0.7106	0.6184	2.752	2.568	2.186
				0.4	0.2487	0.0914	0.1598	1.145	0.6178	2.617	2.356	2.018
				0.6	0.3000	0.0484	0.1793	1.470	0.6041	2.551	2.230	1.902
				0.6950	0.3219	0.0	0.1865	1.625	0.5963	2.529	2.177	1.807
0.4	0.5	0	1	0.15	0.5714	0.3318	0.4824	0.4229	0.3987	1.499	1.349	1.090
				0.7815	0.8782	0.0	0.6203	1.149	0.4135	1.659	1.306	0.7155
		-0.2	1	0.15	0.4377	0.2432	0.3499	0.4634	0.4479	1.749	1.568	1.300
				0.7522	0.7383	0.0	0.4865	1.202	0.4509	1.847	1.448	0.9953
		-0.4	1	0.15	0.3179	0.1653	0.2330	0.5129	0.5069	2.078	1.857	1.582
				0.7172	0.6060	0.0	0.3638	1.257	0.4946	2.072	1.623	1.296

Table 3, cont.

β	ω	f_w	σ_1	t_w	$f''(0)$	$\theta'(0)$	$g'(0)$	I_1	I_2	$I_1(1)$	$I_1(2)$	$I_1(3)$
0.4	0.5	-0.6	1	0.15	0.2160	0.1006	0.1363	0.5746	0.5780	2.521	2.250	1.971
				0.6749	0.4828	0.0	0.2548	1.313	0.5464	2.347	1.842	1.628
	0.7	0	0	0.15	0.5027	0.3597	0.4129	0.3347	0.3687	1.443	1.358	1.204
				1.0	0.8544	0.4632	0.5300	1.219	0.3667	1.521	1.219	1.257
			0.5	0.15	0.5281	0.3422	0.4344	0.3749	0.3781	1.475	1.366	1.166
				0.9058	0.8470	0.0	0.5430	1.185	0.3728	1.551	1.235	0.5240
			1	0.15	0.5807	0.3363	0.4853	0.4358	0.4063	1.546	1.393	1.126
				0.7974	0.8490	0.0	0.5763	1.151	0.3887	1.617	1.265	0.5624
		-0.2	0	0.15	0.3786	0.2662	0.2851	0.3934	0.4211	1.730	1.608	1.429
				1.0	0.7377	0.3727	0.4052	1.346	0.3993	1.720	1.346	1.408
			0.5	0.15	0.4002	0.2511	0.3054	0.4259	0.4298	1.753	1.608	1.391
				0.8880	0.7202	0.0	0.4156	1.280	0.4083	1.754	1.373	0.8373
			1	0.15	0.4468	0.2472	0.3531	0.4760	0.4559	1.800	1.615	1.340
				0.7647	0.7109	0.0	0.4452	1.207	0.4281	1.816	1.416	0.8903
		-0.4	0	0.15	0.2707	0.1829	0.1768	0.4657	0.4845	2.113	1.940	1.735
				1.0	0.6321	0.2903	0.2956	1.491	0.4357	1.954	1.491	1.588
			0.5	0.15	0.2875	0.1706	0.1944	0.4882	0.4923	2.122	1.930	1.696
				0.8656	0.6025	0.0	0.3022	1.383	0.4495	1.996	1.540	1.171
			1	0.15	0.3263	0.1687	0.2362	0.5251	0.5155	2.133	1.906	1.624
				0.7256	0.5810	0.0	0.3264	1.266	0.4741	2.060	1.602	1.223
		-0.6	0	0.15	0.1826	0.1124	0.0933	0.5537	0.5611	2.632	2.391	2.162
				1.0	0.5384	0.2175	0.2034	1.656	0.4760	2.225	1.656	1.800

Table 3, cont.

β	ω	f_w	σ_1	t_w	$f''(0)$	$\theta'(0)$	$g'(0)$	I_1	I_2	$I_1(1)$	$I_1(2)$	$I_1(3)$
0.4	0.7	-0.6	0.5	0.4	0.3251	0.1308	0.1580	0.9262	0.5546	2.449	2.081	1.925
				0.6	0.4078	0.1207	0.1830	1.193	0.5308	2.364	1.913	1.801
				0.8367	0.4950	0.0	0.2054	1.493	0.4978	2.286	1.743	1.531
			1	0.15	0.2228	0.1031	0.1391	0.5857	0.5872	2.579	2.299	2.015
				0.4	0.3517	0.1018	0.1914	0.9521	0.5692	2.454	2.052	1.834
				0.6	0.4316	0.0472	0.2151	1.222	0.5415	2.384	1.894	1.680
				0.6772	0.4601	0.0	0.2226	1.323	0.5298	2.360	1.838	1.596
0.5	0.7	0	0	0.15	0.5243	0.3625	0.4164	0.3182	0.3652	1.433	1.334	1.196
				1.0	0.9277	0.4705	0.5390	1.155	0.3503	1.500	1.155	1.240
		-0.2	0	0.15	0.4014	0.2699	0.2895	0.3704	0.4163	1.711	1.572	1.414
				1.0	0.8126	0.3810	0.4153	1.266	0.3796	0.1689	1.013	1.014
		-0.4	0	0.15	0.2942	0.1874	0.1822	0.4331	0.4777	2.077	1.883	1.706
				1.0	0.7077	0.2995	0.3065	1.391	0.4121	1.909	1.391	1.553
		-0.6	0	0.15	0.2057	0.1178	0.0990	0.5079	0.5518	2.562	2.297	2.105
				0.4	0.3534	0.1682	0.1511	0.8354	0.5357	2.379	1.997	1.936
				0.6	0.4482	0.1927	0.1771	1.077	0.5096	2.289	1.820	1.857
				1.0	0.6138	0.2274	0.2146	1.532	0.4477	2.162	1.532	1.750
		-0.8	0	0.5	0.3255	0.1216	0.0968	1.081	0.5853	2.718	2.186	2.211
				1.0	0.5314	0.1656	0.1410	1.689	0.4866	2.447	1.689	1.979
		-1.0	0	0.15	0.0939	0.0284	0.0140	0.6933	0.7472	4.045	3.601	3.421
				1.0	0.4604	0.1148	0.0859	1.863	0.5489	2.763	1.863	2.240
		-1.2	0	0.15	0.0683	0.0099	0.0031	0.7968	0.8700	5.060	4.531	4.393
				0.4	0.1774	0.0341	0.0169	1.218	0.7683	3.872	3.160	3.236

Table 3, cont.

β	ω	f_w	σ_1	t_w	$f''(0)$	$\theta'(0)$	$g'(0)$	I_1	I_2	$I_1(1)$	$I_1(2)$	$I_1(3)$
0.5	0.7	-1.2	0	0.6	0.2563	0.0499	0.0281	1.513	0.6998	3.506	2.672	2.896
				1.0	0.4006	0.0753	0.0481	2.052	0.5745	3.105	2.052	2.536
		-1.4	0	1.0	0.3510	0.0464	0.0244	2.256	0.6229	3.467	2.256	2.862
0.75	0.7	0	0.5	0.15	0.6029	0.3512	0.4478	0.3187	0.3690	1.446	1.287	1.139
				0.9004	1.072	0.0	0.5719	1.0000	0.3215	1.496	1.044	0.4385
		-0.2	0.5	0.15	0.4771	0.2618	0.3213	0.3535	0.4119	1.701	1.496	1.344
				0.8808	0.9440	0.0	0.4471	1.059	0.3495	1.674	1.146	0.7242
		-0.4	0.5	0.15	0.3651	0.1830	0.2127	0.3941	0.4696	2.027	1.764	1.612
				0.8560	0.8222	0.0	0.3354	1.120	0.3825	1.882	1.267	1.022
		-0.6	0.5	0.15	0.2697	0.1171	0.1256	0.4411	0.5389	2.446	2.111	1.965
				0.4	0.4568	0.1455	0.1844	0.7349	0.5076	2.284	1.798	1.772
				0.6	0.5817	0.1311	0.2134	0.9500	0.4693	2.200	1.604	1.635
				0.8240	0.7064	0.0	0.2387	1.179	0.4221	2.126	1.415	1.338
1	0.5	0	0	0.15	0.5612	0.3317	0.3844	0.2437	0.3176	1.300	1.175	1.096
				1.0	1.233	0.4959	0.5705	0.9402	0.2923	1.430	0.9402	1.186
		-0.2	0	0.15	0.4461	0.2441	0.2633	0.2787	0.3674	1.570	1.394	1.312
				1.0	1.121	0.4090	0.4497	1.009	0.3119	1.593	1.009	1.310
		-0.4	0	0.15	0.3469	0.1684	0.1641	0.3189	0.4279	1.917	1.675	1.595
				1.0	1.018	0.3298	0.3428	1.085	0.3330	1.778	1.085	1.452
		-0.6	0	0.4	0.5028	0.1763	0.1621	0.6238	0.4791	2.174	1.693	1.783
				0.6	0.6559	0.2106	0.1987	0.8128	0.4431	2.094	1.496	1.709
				1.0	0.9226	0.2591	0.2512	1.167	0.3556	1.984	1.167	1.612

Table 3, cont.

β	ω	f_w	σ_1	t_w	$f''(0)$	$\theta'(0)$	$g'(0)$	I_1	I_2	$I_1(1)$	$I_1(2)$	$I_1(3)$
1	0.7	0	0	0.15	0.6146	0.3729	0.4295	0.2614	0.3532	1.396	1.253	1.167
				1.0	1.233	0.4959	0.5705	0.9402	0.2923	1.430	0.9402	1.186
		-0.2	0	0.15	0.4947	0.2828	0.3053	0.2944	0.4004	1.647	1.454	1.364
				1.0	1.121	0.4090	0.4497	1.009	0.3119	1.593	1.009	1.310
		-0.4	0	0.15	0.3886	0.2028	0.2002	0.3320	0.4566	1.965	1.707	1.617
				1.0	1.018	0.3298	0.3428	1.085	0.3330	1.778	1.085	1.452
		-0.6	0	0.15	0.2983	0.1350	0.1175	0.3744	0.5239	2.366	2.028	1.945
				0.4	0.5189	0.1912	0.1770	0.6296	0.4894	2.197	1.704	1.791
				0.6	0.6647	0.2191	0.2072	0.8166	0.4481	2.109	1.503	1.716
				1.0	0.9225	0.2591	0.2512	1.167	0.3556	1.984	1.167	1.612
		-0.8	0	1.0	0.8354	0.1976	0.1758	1.255	0.3799	2.212	1.255	1.793
		-1.2	0	1.0	0.6858	0.1034	0.0731	1.452	0.4330	2.723	1.452	2.218*
		-1.4	0	1.0	0.6229	0.0704	0.0429	1.561	0.4620	3.000	1.561	2.460*
		-1.6	0	1.0	0.5674	0.0457	0.0234	1.675	0.4924	3.287	1.675	2.721*
	1	0	0	0.15	0.7104	0.4473	0.5107	0.2934	0.4171	1.572	1.395	1.296
				1.0	1.233	0.4959	0.5705	0.9402	0.2923	1.430	0.9402	1.186
		-0.2	0	0.15	0.5842	0.3536	0.3823	0.3236	0.4607	1.798	1.572	1.469
				1.0	1.121	0.4090	0.4497	1.009	0.3119	1.593	1.009	1.310
		-0.4	0	0.15	0.4688	0.2677	0.2691	0.3575	0.5112	2.076	1.788	1.682
				1.0	1.018	0.3298	0.3428	1.085	0.3330	1.778	1.085	1.452

*Convergence to 10^{-4}.

Table 3, cont.

β	ω	f_w	σ_1	t_w	$f''(0)$	$\theta'(0)$	$g'(0)$	I_1	I_2	$I_1(1)$	$I_1(2)$	$I_1(3)$
1	1	-0.6	0	0.15	0.3658	0.1912	0.1740	0.3957	0.5702	2.419	2.054	1.951
				1.0	0.9226	0.2591	0.2512	1.167	0.3556	1.984	1.167	1.612
		-0.8	0	0.15	0.2774	0.1260	0.0999	0.4383	0.6392	2.842	2.388	2.294
				1.0	0.8354	0.1976	0.1758	1.255	0.3799	2.212	1.255	1.793
		-1.0	0	0.15	0.2057	0.0742	0.0485	0.4856	0.7201	3.366	2.813	2.738
		-1.2	0	0.15	0.1524	0.0374	0.0187	0.5372	0.8151	4.007	3.351	3.311
				1.0	0.6858	0.1034	0.0731	1.452	0.4330	2.723	1.452	2.218*
		1.4	0	0.15	0.1171	0.0155	0.0054	0.5923	0.9248	4.764	4.012	4.023
				1.0	0.6229	0.0704	0.0429	1.561	0.4620	3.000	1.561	2.460*
		-1.6	0	0.15	0.0960	0.0052	0.0011	0.6497	1.046	5.602	4.764	4.840
				1.0	0.5674	0.0457	0.0234	1.675	0.4924	3.287	1.675	2.721*
2	0.5	0	0	0.4	1.043	0.4276	0.4956	0.3617	0.3196	1.352	1.040	1.131
				0.6	1.276	0.4677	0.5414	0.4884	0.2988	1.367	0.9445	1.140
				1.0	1.687	0.5235	0.6052	0.7282	0.2308	1.363	0.7282	1.134
		-0.4	0	0.6	1.065	0.3048	0.3174	0.5552	0.3544	1.727	1.123	1.420*
				1.0	1.475	0.3605	0.3802	0.8100	0.2549	1.664	0.8100	1.365
	0.7	-0.2	0	0.6	1.180	0.3930	0.4336	0.5248	0.3304	1.557	1.039	1.286*
				1.0	1.578	0.4384	0.4863	0.7678	0.2425	1.506	0.7678	1.243
		-0.4	0	0.4	0.9597	0.2038	0.4094	0.4792	0.3915	1.889	1.226	1.245
				0.6	1.181	0.0984	0.4464	0.6222	0.3327	1.848	1.036	1.035
		-0.6	0	0.15	0.4291	0.1520	0.1364	0.2640	0.5010	2.206	1.807	1.815
				0.4	0.7594	0.2140	0.2032	0.4556	0.4481	2.047	1.459	1.673

*Convergence to 10^{-4}.

Table 3, cont.

β	ω	f_w	σ_1	t_w	$f''(0)$	$\theta'(0)$	$g'(0)$	I_1	I_2	$I_1(1)$	$I_1(2)$	$I_1(3)$
2	0.7	-0.6	0	0.6	0.9815	0.2452	0.2377	0.5948	0.3915	1.962	1.235	1.601 *
				1.0	1.377	0.2904	0.2882	0.8546	0.2679	1.840	0.8546	1.501 *
5	0.5	-0.2	1	0.6	2.107	2.175	0.8352	0.3116	0.2112	0.6670	0.7727	-0.6753**
		-0.6	1	0.4	1.483	0.2409	0.5461	0.3309	0.3575	1.877	1.027	1.159 **

*Convergence to 10^{-4}.

**Convergence to 10^{-3}.

This table is summarized on pages 34--36.

Table 4

SIMILAR SOLUTIONS FOR $t_s = 1$, $f_w = 0$

β	α	σ_1	Pr	t_w	$f''(0)$	$\theta'(0)$	$g'(0)$	I_1	I_2	$I_1(1)$	$I_1(2)$	$I_1(3)$
0	0.5	0	0.7	0.15	0.3490	0.3034	0.3490	0.4037	0.3490	1.411	1.411	1.185
				0.4	0.4143	0.3625	0.4143	0.7886	0.4144	1.571	1.571	1.303
				0.6	0.4399	0.3861	0.4399	1.089	0.4399	1.626	1.626	1.343
				1.0	0.4696	0.4139	0.4696	1.686	0.4696	1.686	1.686	1.385
		0.5	0.5	0.15	0.3816	0.2443	0.3816	0.3951	0.3816	1.478	1.478	1.274
				0.4	0.4437	0.2608	0.4437	0.8234	0.4437	1.623	1.623	1.332
				0.6	0.4662	0.2287	0.4662	1.163	0.4662	1.670	1.670	1.268
				0.8482	0.4834	0.0	0.4834	1.584	0.4834	1.705	1.705	0.7940
			0.7	0.15	0.3734	0.2963	0.3734	0.4608	0.3735	1.452	1.452	1.166
				0.4	0.4394	0.3341	0.4394	0.8600	0.4394	1.610	1.610	1.250
				0.6	0.4642	0.3246	0.4642	1.167	0.4642	1.663	1.663	1.240
				0.9151	0.4876	0.0	0.4876	1.645	0.4876	1.710	1.710	0.7665
		1	0.5	0.15	0.4532	0.2166	0.4532	0.4784	0.4315	1.532	1.532	1.240
				0.4	0.5043	0.1716	0.5043	0.9359	0.4949	1.687	1.687	1.252
				0.6	0.5232	0.0450	0.5232	1.251	0.5106	1.721	1.721	1.174
			0.7	0.05	0.3507	0.2527	0.3507	0.3514	0.3494	1.304	1.304	1.003
				0.15	0.4328	0.3026	0.4328	0.5519	0.4303	1.523	1.523	1.142
				0.4	0.4987	0.3045	0.4987	0.9513	0.4924	1.672	1.672	1.201
				0.6	0.5208	0.2392	0.5208	1.251	0.5124	1.717	1.717	1.163
				0.8009	0.5343	0.0	0.5343	1.546	0.5239	1.742	1.742	0.9846
				1.1	0.5491	1.521	0.5491	1.953	0.5301	1.753	1.753	2.000

Table 4, cont.

β	ω	σ_1	Pr	t_w	f"(0)	θ'(0)	g'(0)	I_1	I_2	$I_1(1)$	$I_1(2)$	$I_1(3)$
0	0.5	all	1	0.15	0.4223	0.4223	0.4223	0.5723	0.4146	1.466	1.466	1.051
				0.4	0.4988	0.4988	0.4988	0.9284	0.4734	1.611	1.611	1.137
				0.6	0.5264	0.5264	0.5264	1.192	0.4939	1.657	1.657	1.163
				1.0	0.5541	0.5541	0.5541	1.698	0.5179	1.709	1.709	1.192
				1.1	0.5550	0.5550	0.5550	1.871	0.5345	1.749	1.749	1.215
	0.7	0	0.7	0.15	0.3920	0.3428	0.3920	0.4414	0.3921	1.508	1.508	1.255
				0.4	0.4353	0.3820	0.4353	0.8137	0.4353	1.614	1.614	1.334
				0.6	0.4514	0.3969	0.4514	1.106	0.4514	1.649	1.649	1.359
				1.0	0.4696	0.4139	0.4696	1.686	0.4696	1.686	1.686	1.385
		all	1	0.6	0.4498	0.4498	0.4498	1.167	0.4498	1.645	1.645	1.196
				1.0	0.4696	0.4696	0.4696	1.686	0.4696	1.686	1.686	1.217
		0.5	0.7	0.15	0.4089	0.3250	0.4089	0.4970	0.4089	1.538	1.538	1.224
				0.4	0.4512	0.3431	0.4512	0.8778	0.4512	1.640	1.640	1.270
				0.6	0.4663	0.3263	0.4663	1.173	0.4663	1.672	1.672	1.248
				0.9162	0.4802	0.0	0.4802	1.656	0.4802	1.700	1.700	0.7691
		1	0.5	0.15	0.4517	0.2254	0.4517	0.5519	0.4512	1.610	1.610	1.245
				0.4	0.4854	0.1793	0.4854	0.9694	0.4844	1.690	1.690	1.201
				0.6	0.4954	0.0733	0.4954	1.294	0.4941	1.712	1.712	1.044
				0.6729	0.4978	0.0	0.4978	1.412	0.4965	1.717	1.717	0.9324
			0.7	0.05	0.3924	0.2843	0.3924	0.3963	0.3923	1.442	1.442	1.100
				0.15	0.4451	0.3136	0.4451	0.5766	0.4448	1.586	1.586	1.188
				0.4	0.4835	0.3025	0.4835	0.9637	0.4832	1.681	1.681	1.195
				0.6	0.4960	0.2420	0.4960	1.253	0.4947	1.707	1.707	1.134
				0.8187	0.5034	0.0	0.5034	1.580	0.5029	1.725	1.725	0.8037
				1.1	0.5097	1.291	0.5097	1.982	0.5085	1.737	1.737	2.444

Table 4, cont.

β	α	σ_1	Pr	t_w	$f''(0)$	$\theta'(0)$	$g'(0)$	I_1	I_2	$I_1(1)$	$I_1(2)$	$I_1(3)$
0	0.7	all	1	0.15	0.4386	0.4386	0.4386	0.6046	0.4369	1.555	1.555	1.118
	1	all	1	0	0.4696	0.4696	0.4696	0.4696	0.4696	1.686	1.686	1.217
				1.0	0.4696	0.4696	0.4696	1.686	0.4696	1.686	1.686	1.217
		0	0.7	0.15	0.4696	0.4139	0.4696	0.5091	0.4696	1.686	1.686	1.385
				0.4	0.4696	0.4139	0.4696	0.8554	0.4696	1.686	1.686	1.385
				0.6	0.4696	0.4139	0.4696	1.132	0.4696	1.686	1.686	1.385
				1.0	0.4696	0.4139	0.4696	1.686	0.4696	1.686	1.686	1.385
		0.5	0.5	0.15	0.4696	0.3029	0.4696	0.4671	0.4696	1.686	1.686	1.434
				0.4	0.4696	0.2763	0.4696	0.8593	0.4696	1.686	1.686	1.378
				0.6	0.4696	0.2311	0.4696	1.173	0.4696	1.686	1.686	1.283
				0.8522	0.4696	0.0	0.4696	1.569	0.4696	1.686	1.686	0.7952
			0.7	0.15	0.4696	0.3739	0.4696	0.5595	0.4696	1.686	1.686	1.326
				0.4	0.4696	0.3572	0.4696	0.9057	0.4696	1.686	1.686	1.301
				0.6	0.4699	0.3281	0.4699	1.177	0.4684	1.683	1.683	1.266
				0.9179	0.4696	0.0	0.4696	1.623	0.4696	1.686	1.686	0.7732
		1	0.5	0.15	0.4696	0.2301	0.4696	0.5812	0.4696	1.686	1.686	1.300
				0.4	0.4696	0.1860	0.4696	0.9738	0.4696	1.686	1.686	1.188
				0.6	0.4696	0.0955	0.4696	1.287	0.4696	1.686	1.686	0.9983
			0.7	0.15	0.4696	0.3339	0.4696	0.6097	0.4696	1.686	1.686	1.267
				0.4	0.4696	0.3006	0.4696	0.9560	0.4696	1.686	1.686	1.217
				0.6	0.4696	0.2439	0.4696	1.233	0.4696	1.686	1.686	1.134
				0.8357	0.4696	0.0	0.4696	1.559	0.4696	1.686	1.686	0.7729
		all	1	0.15	0.4696	0.4696	0.4696	0.6521	0.4696	1.686	1.686	1.217

Table 4, cont.

β	ω	σ_1	Pr	t_w	f"(0)	θ'(0)	g'(0)	I_1	I_2	$I_1(1)$	$I_1(2)$	$I_1(3)$
0.05	1	all	1	0	0.4848	0.4733	0.4733	0.4453	0.4626	1.675	1.654	1.208
				0.5	0.5082	0.4773	0.4773	1.023	0.4570	1.663	1.623	1.199
				1.0	0.5311	0.4812	0.4812	1.593	0.4514	1.652	1.593	1.191
0.1	0.5	1	0.7	0.05	0.3780	0.2597	0.3623	0.3187	0.3414	1.291	1.260	0.9912
				0.15	0.4733	0.3112	0.4476	0.5088	0.4197	1.513	1.464	1.124
				0.4	0.5655	0.3134	0.5194	0.8753	0.4717	1.652	1.575	1.166
				0.6	0.6066	0.2435	0.5450	1.149	0.4840	1.690	1.591	1.106
				0.7948	0.6376	0.0	0.5613	1.409	0.4882	1.709	1.590	0.8825
	0.7	1	0.7	0.05	0.4202	0.2900	0.4021	0.3614	0.3834	1.430	1.394	1.087
				0.15	0.4850	0.3206	0.4573	0.5303	0.4320	1.571	1.521	1.166
				0.4	0.5492	0.3096	0.5008	0.8823	0.4608	1.654	1.576	1.157
				0.6	0.5808	0.2463	0.5164	1.150	0.4655	1.674	1.575	1.063
				0.8120	0.6086	0.5096	0.5274	1.428	0.4653	1.682	1.561	0.7109
				1.1	0.6420	1.492	0.5447	1.659	0.4751	1.702	1.569	0.8969
	1	all	1	0.15	0.5123	0.4788	0.4788	0.5909	0.4531	1.658	1.607	1.196
				0.4	0.5347	0.4825	0.4825	0.8668	0.4480	1.647	1.595	1.188
				0.6	0.5523	0.4854	0.4854	1.085	0.4438	1.639	1.558	1.182
				1.0	0.5870	0.4909	0.4909	1.516	0.4355	1.624	1.516	1.170
		1	0.7	0.15	0.5085	0.3381	0.4781	0.5563	0.4529	1.660	1.610	1.240
				0.4	0.5331	0.3051	0.4824	0.8715	0.4460	1.648	1.576	1.174
				0.6	0.5525	0.2459	0.4857	1.121	0.4404	1.638	1.549	1.071
				0.8290	0.5744	0.0	0.4894	1.403	0.4341	1.627	1.520	0.6798

Table 4, cont.

β	ω	σ_1	Pr	t_w	f"(0)	θ'(0)	g'(0)	I_1	I_2	$I_1(1)$	$I_1(2)$	$I_1(3)$
0.15	0.7	0	0.7	0.6	1.145	0.4703	0.5420	0.5504	0.3196	1.415	1.021	1.176
0.2	0.5	1	0.7	0.05	0.4015	0.2657	0.3718	0.2977	0.3366	1.285	1.230	0.9808
				0.15	0.5091	0.3188	0.4604	0.4752	0.4114	1.507	1.419	1.110
				0.4	0.6243	0.3209	0.5370	0.8161	0.4550	1.640	1.499	1.139
				0.6	0.6818	0.2467	0.5655	1.069	0.4608	1.673	1.494	1.062
				0.7897	0.7268	0.0	0.5838	1.303	0.4589	1.686	1.474	0.8130
				1.1	0.7913	1.748	0.6069	1.659	0.4479	1.680	1.425	2.337
	0.7	1	0.7	0.05	0.4445	0.2950	0.4103	0.3357	0.3766	1.422	1.358	1.076
				0.15	0.5202	0.3265	0.4678	0.4925	0.4216	1.560	1.470	1.150
				0.4	0.6068	0.3147	0.5155	0.8141	0.4426	1.633	1.495	1.136
				0.6	0.6548	0.2470	0.5338	1.059	0.4415	1.646	1.473	1.036
				0.8064	0.6979	0.0	0.5465	1.314	0.4350	1.653	1.439	0.6471
				1.1	0.7539	1.489	0.5607	1.657	0.4223	1.647	1.386	2.709
	1	all	1	0	0.5233	0.4821	0.4821	0.3881	0.4457	1.648	1.576	1.188
				0.5	0.6070	0.4951	0.4951	0.8995	0.4271	1.612	1.480	1.161
				1.0	0.6867	0.5069	0.5069	1.392	0.4082	1.580	1.392	1.138
		1	0.7	0.15	0.5424	0.3414	0.4851	0.5143	0.4395	1.639	1.550	1.219
				0.4	0.5883	0.3085	0.4926	0.8065	0.4270	1.618	1.491	1.141
				0.8235	0.6633	0.0	0.5044	1.288	0.4057	1.586	1.397	0.6165
0.2857	0.5	0	0.7	0.15	0.4283	0.3157	0.3643	0.3292	0.3342	1.360	1.302	1.144
				1.0	0.7627	0.4534	0.5179	1.309	0.3887	1.551	1.309	1.280

Table 4, cont.

β	ω	σ_1	Pr	t_w	$f''(0)$	$\theta'(0)$	$g'(0)$	I_1	I_2	$I_1(1)$	$I_1(2)$	$I_1(3)$
0.2857	0.5	0.5	0.7	0.15	0.4605	0.3101	0.3934	0.3742	0.3537	1.406	1.328	1.123
				0.9071	0.7683	0.0	0.5397	1.282	0.4019	1.595	1.335	0.5663
		1	0.7	0.05	0.4209	0.2710	0.3808	0.2779	0.3299	1.268	1.199	0.9696
				0.15	0.5371	0.324[illegible]	0.4704	0.4503	0.4053	1.502	1.385	1.100
				0.4	0.6701	0.326[illegible]	0.5505	0.7737	0.4428	1.631	1.446	1.120
				0.6	0.7401	0.2490	0.5810	1.013	0.4439	1.661	1.426	1.032
				0.7849	0.7952	0.0	0.6012	1.223	0.4374	1.667	1.393	0.7890
				1.1	0.8787	1.862	0.6294	1.[illegible]3	0.4195	1.640	1.325	2.232
	0.7	1	0.7	0.05	0.4636	0.2988	0.4171	0.3143	0.3707	1.411	1.329	1.068
				0.15	0.5475	0.3310	0.4758	0.4657	0.4143	1.553	1.433	1.138
				0.4	0.6512	0.3194	0.5259	0.7744	0.4298	1.624	1.439	1.108
				0.6	0.7118	0.2499	0.5459	1.007	0.4241	1.635	1.402	0.9885
				0.8022	0.7664	0.0	0.5603	1.236	0.4133	1.635	1.356	0.6055
				1.1	0.8401	1.565	0.5771	1.557	0.3950	1.621	1.286	2.709
	1	all	1	0	0.5419	0.4862	0.4862	0.3629	0.4332	1.636	1.542	1.179
				0.2	0.5883	0.4932	0.4932	0.5593	0.4285	1.617	1.491	1.165
				0.4	0.6334	0.4998	0.4998	0.7517	0.4186	1.599	1.443	1.151
				0.6	0.6774	0.5061	0.5061	0.9406	0.4087	1.582	1.396	1.139
				0.8	0.7205	0.5121	0.5121	1.126	0.3987	1.566	1.352	1.127
				1.0	0.[illegible]	0.5179	0.5179	1.309	0.3887	1.551	1.339	1.116
	1	1	0.7	0.15	0.5684	0.3438	0.490[illegible]	0.4846	0.4299	1.625	1.508	1.204
				0.4	0.6308	0.3108	0.4999	0.7611	0.4134	1.598	1.432	1.117

Table 4, cont.

β	ω	σ_1	Pr	τ_w	$f''(0)$	$\theta'(0)$	$g'(0)$	I_1	I_2	$I_1(1)$	$I_1(2)$	$I_1(3)$
0.2857	1	1	0.7	0.6	0.6793	0.2473	0.5072	0.9773	0.4001	1.578	1.374	0.9918
				0.8194	0.7310	0.0	0.5148	1.210	0.3854	1.558	1.314	0.5758
0.3	1	all	1	0	0.5448	0.4868	0.4868	0.3591	0.4371	1.634	1.537	1.178
				0.2	0.5931	0.4941	0.4941	0.5539	0.4270	1.614	1.484	1.163
				0.4	0.6402	0.5009	0.5009	0.7446	0.4168	1.596	1.434	1.149
				0.6	0.6860	0.5074	0.5074	0.9318	0.4065	1.578	1.386	1.136
				0.8	0.7309	0.5136	0.5136	1.116	0.3961	1.562	1.341	1.124
				1.0	0.7748	0.5195	0.5195	1.297	0.3857	1.547	1.297	1.113
				2.0	0.9829	0.5457	0.5457	2.165	0.3334	1.484	1.099	1.066
0.4	0.5	0	0.7	0.15	0.4540	0.3192	0.3686	0.6094	0.3303	1.347	1.273	1.133
				1.0	0.8544	0.4632	0.5300	1.219	0.3667	1.521	1.219	1.257
		0.5	0.7	0.15	0.4887	0.3141	0.3992	0.3511	0.3483	1.394	1.296	1.111
				0.9048	0.8564	0.0	0.5533	1.194	0.3786	1.570	1.244	0.5232
		1	0.7	0.05	0.4416	0.2757	0.3870	0.2694	0.3321	1.289	1.188	0.9672
				0.15	0.5714	0.3318	0.4824	0.4229	0.3987	1.499	1.349	1.090
				0.4	0.7260	0.3334	0.5667	0.7270	0.4291	1.624	1.386	1.099
				0.6	0.8112	0.2515	0.5995	0.9516	0.4246	1.651	1.351	0.9983
				0.7815	0.8782	0.0	0.6203	1.149	0.4135	1.659	1.306	0.7155
				1.1	0.9837	1.924	0.6189	1.474	0.3884	1.644	1.218	2.562
	0.7	0	0.7	0.15	0.5027	0.3597	0.4129	0.3347	0.3687	1.443	1.358	1.204
				1.0	0.8544	0.4632	0.5300	1.219	0.3667	1.521	1.219	1.257
		0.5	0.7	0.15	0.5281	0.3422	0.4344	0.3749	0.3781	1.475	1.366	1.166
				0.4	0.6583	0.3671	0.4909	0.6569	0.3955	1.544	1.362	1.176

Table 4, cont.

β	ω	σ_1	Pr	t_w	$f''(0)$	$\theta'(0)$	$g'(0)$	I_1	I_2	$I_1(1)$	$I_1(2)$	$I_1(3)$
0.4	0.7	0.5	0.7	0.6	0.7386	0.3500	0.5159	0.8701	0.3906	1.554	1.318	1.119
				0.9058	0.8470	0.0	0.5430	1.185	0.3728	1.551	1.235	0.5240
		1	0.7	0.05	0.4864	0.3034	0.4246	0.2943	0.3654	1.404	1.300	1.058
				0.15	0.5807	0.3363	0.4853	0.4358	0.4063	1.546	1.393	1.126
				0.4	0.7053	0.3239	0.5389	0.7220	0.4154	1.608	1.377	1.092
				0.6	0.7811	0.2500	0.5609	0.9380	0.4050	1.614	1.326	0.9700
				0.7974	0.8490	0.0	0.5763	1.151	0.3887	1.617	1.265	0.5624
				1.1	0.9451	1.536	0.5953	1.460	0.3616	1.603	1.172	2.887
	1	all	1	0	0.5639	0.4908	0.4908	0.3350	0.4299	1.623	1.505	1.170
				0.2	0.6254	0.4997	0.4997	0.5198	0.4175	1.599	1.441	1.152
				0.4	0.6850	0.5079	0.5079	0.7001	0.4050	1.577	1.381	1.135
				0.6	0.7429	0.5157	0.5157	0.8765	0.3923	1.557	1.325	1.120
				0.8	0.7993	0.5231	0.5231	1.049	0.3795	1.538	1.271	1.107
				1.0	0.8544	0.5300	0.5300	1.219	0.3667	1.521	1.219	1.094
		0	0.7	0.15	0.5901	0.4327	0.4926	0.3806	0.4380	1.619	1.514	1.333
				1.0	0.8544	0.4632	0.5300	1.219	0.3667	1.521	1.219	1.257
		0.5	0.7	0.15	0.5948	0.3901	0.4942	0.4156	0.4286	1.614	1.487	1.261
				0.9074	0.8337	0.0	0.5283	1.173	0.3646	1.525	1.221	0.5248
		1	0.7	0.15	0.5998	0.3465	0.4959	0.4515	0.4192	1.608	1.460	1.187
				0.4	0.6824	0.3134	0.5082	0.7109	0.3980	1.575	1.366	1.092
				0.6	0.7461	0.2474	0.5173	0.9129	0.3809	1.552	1.295	0.9564
				0.8147	0.8125	0.0	0.5264	1.125	0.3624	1.529	1.224	0.5334

Table 4, cont.

β	ω	σ_1	Pr	t_w	$f''(0)$	$\theta'(0)$	$g'(0)$	I_1	I_2	$I_1(1)$	$I_1(2)$	$I_1(3)$
0.5	0.5	0	0.5	0.15	0.4646	0.2823	0.3746	0.2303	0.3495	1.372	1.324	1.286
				0.4	0.6386	0.3456	0.4546	0.5099	0.3837	1.476	1.342	1.387
				0.6	0.7451	0.3742	0.4905	0.7282	0.3809	1.496	1.292	1.410
			0.7	0.05	0.3518	0.2520	0.2923	0.1704	0.2666	1.153	1.103	0.9820
				0.15	0.4747	0.3218	0.3720	0.2946	0.3274	1.336	1.251	1.125
				0.4	0.6472	0.3940	0.4533	0.5628	0.3658	1.455	1.290	1.212
				0.6	0.7511	0.4266	0.4900	0.7652	0.3685	1.484	1.258	1.232
				1.0	0.9277	0.4705	0.5390	1.155	0.3503	1.500	1.155	1.240
				1.1	0.9681	0.4791	0.5486	1.250	0.3431	1.500	1.126	1.240
		all	1	0.15	0.4846	0.3689	0.3689	0.3530	0.3081	1.301	1.185	0.9788
				0.4	0.6558	0.4517	0.4517	0.6109	0.3503	1.433	1.243	1.054
				0.6	0.7572	0.4890	0.4890	0.7989	0.3578	1.471	1.227	1.071
		0.25	0.7	0.05	0.3653	0.2517	0.3038	0.1840	0.2733	1.171	1.112	0.9772
				0.15	0.4909	0.3187	0.3857	0.3127	0.3345	1.358	1.259	1.114
				0.9523	0.9260	0.0	0.5494	1.142	0.3549	1.522	1.165	0.4897
		0.5	0.7	0.05	0.3827	0.2532	0.3189	0.2001	0.2826	1.194	1.124	0.9728
				0.15	0.5114	0.3171	0.4037	0.3339	0.3444	1.385	1.271	1.103
				0.4	0.6867	0.3637	0.4878	0.6147	0.3794	1.507	1.303	1.147
				0.6	0.7908	0.3539	0.5247	0.8245	0.3786	1.537	1.264	1.098
				0.9030	0.9266	0.0	0.5635	1.131	0.3612	1.552	1.179	0.4935
				1.1	1.007	1.024	0.5830	1.325	0.3446	1.553	1.117	2.080
		0.6	0.7	0.05	0.3913	0.2546	0.3265	0.2077	0.2874	1.206	1.130	0.9712
				0.15	0.5215	0.317[illegible]	0.4128	0.3438	0.3496	1.399	1.277	1.098
				0.8826	0.9280	0.0	0.5708	1.127	0.3645	1.566	1.185	0.4953

Table 4, cont.

β	ω	σ_1	Pr	t_w	f''(0)	θ'(0)	g'(0)	I_1	I_2	$I_1(1)$	$I_1(2)$	$I_1(3)$
0.5	0.5	0.8	0.7	0.05	0.4138	0.2603	0.3471	0.2261	0.3007	1.236	1.146	0.9680
				0.1	0.4928	0.2989	0.4028	0.3017	0.3419	1.366	1.249	1.052
				0.15	0.5475	0.3204	0.4371	0.3675	0.3636	1.435	1.294	1.090
				0.8393	0.9335	0.0	0.5906	1.121	0.3731	1.605	1.201	0.4989
		1	0.7	0.05	0.4614	0.2815	0.3970	0.2496	0.3233	1.258	1.155	0.9530
				0.15	0.5998	0.3380	0.4938	0.3980	0.3908	1.479	1.315	1.070
				0.4	0.7723	0.3385	0.5831	0.6808	0.4149	1.586	1.334	1.089
				0.6	0.8688	0.2521	0.6156	0.8997	0.4094	1.629	1.294	0.9863
				0.7762	0.9439	0.0	0.6374	1.080	0.3958	1.634	1.242	0.7265
				1.1	1.069	2.001	0.6666	1.404	0.3640	1.632	1.128	2.655
	0.7	0	0.5	0.15	0.5099	0.3173	0.4161	0.2437	0.3881	1.462	1.408	1.370
			0.7	0.05	0.4252	0.3109	0.3578	0.2008	0.3248	1.307	1.244	1.098
				0.15	0.5243	0.3625	0.4164	0.3182	0.3652	1.433	1.334	1.196
				0.4	0.6713	0.4142	0.4750	0.5779	0.3823	1.499	1.324	1.244
				0.6	0.7643	0.4378	0.5018	0.7751	0.3768	1.508	1.275	1.249
				1.0	0.9277	0.4705	0.5390	1.155	0.3503	1.500	1.155	1.240
				1.1	0.9657	0.4770	0.5464	1.247	0.3420	1.496	1.124	1.237
		all	1	0.4	0.6825	0.4754	0.4754	0.6305	0.3673	1.482	1.279	1.080
				0.6	0.7720	0.5022	0.5022	0.8110	0.3665	1.497	1.245	1.085
				1.0	0.9277	0.5390	0.5390	1.155	0.3503	1.500	1.155	1.078
		0.5	0.7	0.05	0.4500	0.3013	0.3784	0.2322	0.3339	1.339	1.253	1.075
				0.15	0.5513	0.3452	0.4388	0.3560	0.3733	1.465	1.340	1.157
				0.4	0.6986	0.3710	0.4975	0.6238	0.3867	1.530	1.321	1.162
				0.6	0.7913	0.3535	0.5238	0.8258	0.3784	1.538	1.266	1.101
				0.9040	0.9169	0.0	0.5526	1.122	0.3557	1.532	1.170	0.4941
				1.1	0.9920	0.9870	0.5674	1.308	0.3379	1.524	1.105	2.030

Table 4, cont.

β	ω	σ_1	Pr	t_w	$f''(0)$	$\theta'(0)$	$g'(0)$	I_1	I_2	$I_1(1)$	$I_1(2)$	$I_1(3)$
0.5	0.7	1	0.5	0.15	0.6047	0.2377	0.4964	0.4006	0.4034	1.558	1.384	1.157
				0.4	0.7484	0.1805	0.5481	0.7066	0.3951	1.600	1.316	1.015
				0.6	0.8397	0.0407	0.5706	0.9314	0.3745	1.591	1.235	0.7591
			0.7	0.05	0.5048	0.3071	0.4305	0.2794	0.3616	1.400	1.278	1.051
				0.15	0.6073	0.3405	0.4928	0.4134	0.4004	1.540	1.363	1.118
				0.4	0.7489	0.3277	0.5488	0.6862	0.4047	1.599	1.332	1.076
				0.6	0.8370	0.2518	0.5719	0.8938	0.3900	1.607	1.268	0.9366
				0.7934	0.9146	0.0	0.5886	1.087	0.3711	1.603	1.200	0.5470
				1.1	1.029	1.700	0.6095	1.386	0.3372	1.588	1.090	2.953
	1	all	1	0	0.5811	0.4942	0.4942	0.3146	0.4238	1.613	1.477	1.162
				0.1	0.6184	0.4995	0.4995	0.4034	0.4167	1.599	1.440	1.152
				0.15	0.6368	0.5020	0.5020	0.4474	0.4131	1.592	1.422	1.147
				0.2	0.6550	0.5045	0.5045	0.4909	0.4095	1.586	1.404	1.142
				0.4	0.7262	0.5140	0.5140	0.6625	0.3949	1.561	1.337	1.124
				0.5	0.7609	0.5185	0.5185	0.7468	0.3876	1.550	1.304	1.115
				0.6	0.7952	0.5228	0.5228	0.8301	0.3802	1.539	1.273	1.107
				0.8	0.8623	0.5311	0.5311	0.9941	0.3653	1.519	1.212	1.092
				1.0	0.9277	0.5390	0.5390	1.155	0.3503	1.500	1.155	1.078
				2.0	1.235	0.5729	0.5729	1.920	0.2746	1.424	0.8988	1.022
		0	0.5	0.15	0.5913	0.3806	0.4907	0.2686	0.2570	1.626	1.561	1.521
				0.4	0.6955	0.3913	0.5067	0.5385	0.4262	1.582	1.430	1.486
				0.6	0.7754	0.3990	0.5183	0.7484	0.4011	1.552	1.333	1.461
				1.0	0.9277	0.4130	0.5390	1.155	0.3503	1.500	1.155	1.419
			0.7	0.15	0.6136	0.4358	0.4964	0.3609	0.4331	1.609	1.487	1.325
				0.4	0.7105	0.4471	0.5103	0.6026	0.4092	1.572	1.380	1.296
				0.6	0.7850	0.4555	0.5205	0.7906	0.3897	1.545	1.301	1.276
				1.0	0.9277	0.4705	0.5390	1.155	0.3503	1.500	1.155	1.240

Table 4, cont.

β	ω	σ_1	Pr	t_w	$f''(0)$	$\theta'(0)$	$g'(0)$	I_1	I_2	$I_1(1)$	$I_1(2)$	$I_1(3)$
0.5	1	1	0.5	0.15	0.6160	0.2440	0.4990	0.4148	0.4086	1.599	1.425	1.189
				0.4	0.7243	0.1831	0.5154	0.6966	0.3760	1.556	1.293	0.9936
				0.6	0.8071	0.0693	0.5273	0.9150	0.3496	1.525	1.195	0.7003
				0.6653	0.8335	0.0	0.5309	0.9852	0.3409	1.516	1.165	0.5362
			0.7	0.15	0.6249	0.3[illegible]	0.5003	0.4272	0.4113	1.596	1.425	1.174
				0.4	0.7237	0.3152	0.5145	0.6743	0.3866	1.559	1.318	1.073
				0.6	0.7997	0.2474	0.5248	0.8662	0.3667	1.533	1.238	0.9303
				0.8111	0.8773	0.0	0.5349	1.064	0.3454	1.508	1.159	0.5038
				1.1	0.9800	1.409	0.5478	1.328	0.3161	1.478	1.058	2.702
0.75	0.5	0.5	0.7	0.05	0.4131	0.2576	0.3251	0.1796	0.2784	1.182	1.092	0.9608
				0.15	0.5618	0.3234	0.4129	0.3000	0.3367	1.368	1.223	1.086
				0.4	0.7768	0.3721	0.5020	0.5518	0.3630	1.483	1.224	1.121
				0.6	0.9092	0.3616	0.5419	0.7391	0.3549	1.506	1.165	1.064
				0.8994	1.083	0.0	0.5840	1.008	0.3264	1.518	1.052	0.4382
				1.1	1.189	1.085	0.6061	1.183	0.3017	1.517	0.9717	2.118
	0.7	0	0.7	0.05	0.45[illegible]7	0.3150	0.3629	0.1800	0.3207	1.292	1.211	1.085
				0.15	0.5726	0.3683	0.4237	0.2857	0.3582	1.412	1.288	1.179
				0.4	0.7568	0.4232	0.4862	0.5183	0.3682	1.470	1.251	1.221
				0.6	0.8770	0.4489	0.5155	0.6939	0.3565	1.474	1.183	1.223
				1.0	1.090	0.4849	0.5568	1.031	0.3174	1.460	1.031	1.209
				1.1	1.140	0.4922	0.5652	1.113	0.3059	1.454	0.9922	1.204

Table 4, cont.

β	ω	σ_1	Pr	ξ_w	f"(0)	θ'(0)	g'(0)	I_1	I_2	$I_1(1)$	$I_1(2)$	$I_1(3)$
0.75	0.7	0.5	0.7	0.05	0.4833	0.3059	0.3850	0.2072	0.3279	1.325	1.215	1.061
				0.15	0.6029	0.3512	0.4478	0.3187	0.3690	1.446	1.287	1.139
				0.4	0.7885	0.3787	0.5108	0.5591	0.3692	1.503	1.240	1.135
				0.6	0.9089	0.3605	0.5400	0.7396	0.3542	1.507	1.166	1.065
				0.9044	1.072	0.0	0.5719	1.0000	0.3215	1.496	1.044	0.4385
				1.1	1.173	1.045	0.5890	1.169	0.2963	1.485	0.9624	2.062
	1	all	1	0	0.6181	0.5012	0.5012	0.2748	0.4120	1.594	1.423	1.148
				0.15	0.6948	0.5112	0.5112	0.3953	0.3984	1.568	1.355	1.129
				0.4	0.8173	0.5262	0.5262	0.5896	0.3751	1.531	1.250	1.101
				0.5	0.8646	0.5318	0.5318	0.6654	0.3656	1.517	1.211	1.091
				0.6	0.9112	0.5371	0.5371	0.7402	0.3560	1.504	1.173	1.081
				1.0	1.090	0.5568	0.5568	[illegible]	0.3174	1.460	1.031	1.048
1	0.5	0	0.5	0.15	0.5440	0.2891	0.3857	0.1968	0.3473	1.340	1.269	1.261
				0.4	0.7923	0.3575	0.4734	0.4248	0.3674	1.428	1.234	1.349
				0.6	0.9521	0.3894	0.5142	0.6005	0.3516	1.439	1.146	1.364
				1.0	1.233	0.4334	0.5705	0.9402	0.2923	1.430	0.9402	1.364
			0.7	0.05	0.4025	0.2584	0.3004	0.1408	0.2617	1.127	1.053	0.9605
				0.15	0.5612	0.3317	0.3844	0.2437	0.3476	1.300	1.175	1.096
				0.4	0.8060	0.4097	0.4730	0.4631	0.3430	1.404	1.167	1.172
				0.6	0.9615	0.4460	0.5140	0.6272	0.3344	1.426	1.102	1.186
				1.0	1.233	0.4959	0.5705	0.9402	0.2923	1.430	0.9402	1.186
				1.1	1.295	0.5058	0.5817	1.016	0.2791	1.429	0.8978	1.184
		all	1	0.15	0.5789	0.3826	0.3826	0.2868	0.2921	1.261	1.092	0.9475
				0.4	0.8204	0.4722	0.4722	0.4985	0.3219	1.380	1.107	1.014
				0.6	0.9714	0.5137	0.5137	0.6518	0.3196	1.411	1.062	1.026
				1.0	1.233	0.5705	0.5705	0.9402	0.2923	1.430	0.9402	1.026

Table 4, cont.

β	ω	σ_1	Pr	t_w	$f''(0)$	$\theta'(0)$	$g'(0)$	I_1	I_2	$I_1(1)$	$I_1(2)$	$I_1(3)$
1	0.5	0.25	0.7	0.05	0.4187	0.2588	0.3132	0.1514	0.2673	1.146	1.059	0.9556
				0.15	0.5808	0.3291	0.3998	0.2578	0.3230	1.323	1.180	1.084
				0.9490	1.224	0.0	0.5829	0.9289	0.2955	1.458	0.9490	0.3943
		0.5	0.7	0.05	0.4395	0.2612	0.3302	0.1641	0.2755	1.172	1.068	0.9516
				0.15	0.6057	0.3285	0.4204	0.2746	0.3311	1.355	1.187	1.074
				0.4	0.8558	0.3787	0.5132	0.5051	0.3507	1.465	1.166	1.102
				0.6	1.013	0.3675	0.5554	0.6760	0.3369	1.488	1.092	1.039
				0.8965	1.219	0.0	0.6000	0.9187	0.3000	1.495	0.9601	0.3992
				1.1	1.349	1.135	0.6241	1.080	0.2689	1.492	0.8652	2.149
		0.6	0.7	0.05	0.4499	0.2632	0.3390	0.1700	0.2799	1.185	1.073	0.9504
				0.1	0.5467	0.3047	0.3954	0.2298	0.3169	1.307	1.159	1.033
				0.15	0.6181	0.3292	0.4310	0.2825	0.3356	1.371	1.192	1.070
				0.8748	1.218	0.0	0.6091	0.9152	0.3025	1.513	0.9655	0.4013
		0.8	0.7	0.05	0.4774	0.2708	0.3632	0.1848	0.2926	1.221	1.086	0.9490
				0.1	0.5774	0.3115	0.4227	0.2472	0.3302	1.348	1.173	1.029
				0.15	0.6501	0.3344	0.4599	0.3016	0.3486	1.415	1.205	1.063
				0.8290	1.218	0.0	0.6344	0.9094	0.3090	1.565	0.9788	0.4061
		1	0.7	0.05	0.5381	0.3014	0.4277	0.2068	0.3133	1.244	1.087	0.9266
				0.15	0.7166	0.3623	0.5342	0.3297	0.3745	1.473	1.221	1.049
				0.4	0.9653	0.3634	0.6465	0.5532	0.3694	1.488	1.163	1.016
				0.6	1.110	0.2598	0.6760	0.7445	0.3577	1.603	1.104	0.8990
				0.7640	1.220	0.0	0.7006	0.8838	0.3329	1.606	1.027	0.6067
				1.1	1.423	2.402	0.7483	1.136	0.2860	1.517	0.8877	2.484
	0.7	0	0.5	0.1	0.5341	0.3060	0.4036	0.1601	0.3739	1.396	1.341	1.312
				0.4	8.151	0.3750	0.4934	0.4327	0.3824	1.468	1.265	1.387
				0.6	0.9645	0.3992	0.5251	0.6061	[illegible]	1.461	1.160	1.385

Table 4, cont.

β	ω	σ_1	Pr	t_w	$f''(0)$	$\theta'(0)$	$g'(0)$	i_1	I_2	$I_1(1)$	$I_1(2)$	$I_1(3)$
1	0.7	0	0.7	0.5	0.4821	0.3183	0.3670	0.1643	0.3177	1.280	1.186	1.076
				0.15	0.6146	0.3729	0.4295	0.2614	0.3532	1.396	1.253	1.168
				0.4	0.8317	0.4302	0.4950	0.4741	0.3576	1.449	1.197	1.205
				0.6	0.9756	0.4574	0.5262	0.6343	0.3413	1.450	1.116	1.203
				1.0	1.233	0.4959	0.5705	0.9402	0.2923	1.430	0.9402	1.186
				1.1	1.293	0.5037	0.5795	1.014	0.2783	1.424	0.8964	1.181
		all	1	0.15	0.6381	0.4310	0.4310	0.3118	0.3262	1.363	1.167	1.007
				0.6	0.9873	0.5271	0.5271	0.6605	0.3268	1.438	1.076	1.040
				1.0	1.233	0.5705	0.5705	0.9402	0.2923	1.430	0.9402	1.026
		0.5	0.7	0.05	0.5120	0.3095	0.3902	0.1884	0.3235	1.314	1.187	1.051
				0.15	0.6478	0.3560	0.4550	0.2908	0.3570	1.432	1.248	1.126
				0.4	1.505	1.024	1.366	0.2288	0.1155	0.4640	0.4537	0.3749
				0.6	1.012	0.3658	0.5526	0.6759	0.3359	1.484	1.092	1.039
				0.8975	1.207	0.0	0.5869	0.9113	0.2955	1.470	0.9523	0.3995
				1.1	1.331	1.091	0.6057	1.067	0.2644	1.457	0.8578	2.089
		0.9	0.7	0.15	0.6940	0.3506	0.4955	0.3239	0.3702	1.491	1.254	1.095
				0.4	0.9117	0.3494	0.5617	0.5505	0.3612	1.544	1.173	1.037
				0.6	1.055	0.2822	0.5919	0.7195	0.3351	1.543	1.075	0.8892
				0.8093	1.192	0.0	0.6160	0.8902	0.3019	1.534	0.9683	0.4096
		1	0.7	0.05	0.5809	0.3222	0.4548	0.2275	0.3488	1.387	1.202	1.026
				0.15	0.7188	0.3576	0.5230	0.3372	0.3810	1.528	1.262	1.088
				0.4	0.9317	0.3426	0.5891	0.5616	0.3668	1.562	1.175	1.023
				0.6	1.071	0.2551	0.6162	0.7336	0.3379	1.579	1.075	0.8527
				0.7779	1.185	0.0	0.6372	0.8753	0.3108	1.549	0.9880	0.5073
				1.1	1.380	1.970	0.6650	1.130	0.2547	1.527	0.8266	3.035

Table 4, cont.

β	ω	σ_1	Pr	t_w	$f''(0)$	$\theta'(0)$	$g'(0)$	I_1	I_2	$I_1(1)$	$I_1(2)$	$I_1(3)$
1	1	0	0.5	0.4	0.8524	0.4037	0.5261	0.4458	0.4066	1.535	1.315	1.449
				0.6	0.9840	0.4145	0.5422	0.6149	0.3691	1.495	1.182	1.417
				1.0	1.233	0.4334	0.5705	0.9402	0.2923	1.430	0.9402	1.364
			0.7	0	0.6071	0.4362	0.4970	0.1695	0.4376	1.607	1.494	1.324
				0.15	0.7104	0.4473	0.5107	0.2934	0.4171	1.572	1.396	1.296
				0.4	0.8735	0.4637	0.5308	0.4921	0.3814	1.521	1.246	1.257
				0.5	0.9362	0.4696	0.5382	0.5692	0.3669	1.504	1.191	1.244
				0.6	0.9976	0.4753	0.5452	0.6454	0.3522	1.487	1.138	1.231
				1.0	1.233	0.4959	0.5705	0.9402	0.2923	1.430	0.9402	1.186
		all	1	0	0.6489	0.5067	0.5067	0.2456	0.4033	1.579	1.383	1.137
				0.15	0.7445	0.5183	0.5183	0.3570	0.3875	1.550	1.305	1.115
				0.2	0.7755	0.5219	0.5219	0.3934	0.3821	1.541	1.280	1.109
				0.4	0.8963	0.5357	0.5357	0.5360	0.3603	1.508	1.186	1.084
				0.5	0.9548	0.5421	0.5421	0.6056	0.3491	1.493	1.142	1.073
				0.6	1.012	0.5482	0.5482	0.6743	0.3380	1.479	1.099	1.063
				0.8	1.124	0.5597	0.5597	0.8089	0.3153	1.454	1.018	1.043
				1.0	1.233	0.5705	0.5705	0.9402	0.2923	1.430	0.9402	1.026
				2.0	1.737	0.6156	0.6156	1.561	0.1761	1.340	0.6010	0.9595

Table 4, cont.

β	ω	σ_1	Pr	t_w	$f''(0)$	$\theta'(0)$	$g'(0)$	I_1	I_2	$I_1(1)$	$I_1(2)$	$I_1(3)$
1.4	0.5	0.6	0.7	0.1	0.5977	0.3107	0.4043	0.2042	0.3118	1.295	1 122	1.020
		0.8	0.7	0.1	0.6320	0.3189	0.4343	0.2195	0.3248	1.341	1.135	1.018
	0.7	0.6	0.7	0.15	0.7200	0.3596	0.4719	0.2640	0.3505	1.426	1.200	1.101
				0.4	0.9874	0.3830	0.5427	0.4622	0.3404	1.472	1.105	1.071
				0.6	1.167	0.3516	0.5765	0.6100	0.3125	1.469	0.9983	0.9707
				0.8721	1.389	0.0	0.6109	0.8028	0.2653	1.453	0.8486	0.3577
				1.1	1.564	1.295	0.6343	0.9587	0.2214	1.437	0.7246	2.341
1.5	0.5	0	0.7	0.05	0.4427	0.2628	0.3060	0.1225	0.2590	1.110	1.022	0.9465
				0.15	0.6304	0.3383	0.3928	0.2123	0.3119	1.277	1.128	1.078
				0.4	0.9337	0.4200	0.4860	0.4030	0.3291	1.374	1.092	1.148
				0.6	1.131	0.4585	0.5298	0.5448	0.3134	1.391	1.008	1.159
				1.0	1.477	0.5119	0.5906	0.8141	0.2562	1.390	0.8141	1.155
		0.25	0.7	0.05	0.4608	0.2636	0.3197	0.1312	0.2640	1.130	1.026	0.9418
				0.15	0.6526	0.3361	0.4095	0.2242	0.3163	1.302	1.130	1.066
				0.9469	1.464	0.0	0.6045	0.8034	0.2585	1.421	0.8216	0.8415
		0.5	0.7	0.05	0.4842	0.2667	0.3381	0.1419	0.2715	1.158	1.033	0.9384
				0.15	0.6810	0.3362	0.4319	0.2383	0.3234	1.336	1.136	1.056
				0.4	0.9917	0.3888	0.5304	0.4389	0.3332	1.440	1.084	1.075
				0.6	1.192	0.3763	0.5760	0.5871	0.3111	1.459	0.9885	1.004
				0.8922	1.453	0.0	0.6241	0.7937	0.2620	1.463	0.8311	0.3466
	0.7	0	0.7	0.05	0.5266	0.3233	0.3733	0.1419	0.3138	1.262	1.150	1.061
				0.15	0.6863	0.3799	0.4383	0.2267	0.3462	1.373	1.203	1.149
				0.4	0.9604	0.4407	0.5082	0.4117	0.3427	1.418	1.120	1.181
				1.0	1.477	0.5119	0.5906	0.8140	0.2561	1.390	0.8140	1.155

Table 4, cont.

β	ω	σ_1	Pr	t_w	$f''(0)$	$\theta'(0)$	$g'(0)$	I_1	I_2	$I_1(1)$	$I_1(2)$	$I_1(3)$
1.5	1	all	1	0	0.6987	0.5147	0.5147	0.2049	0.3914	1.558	1.326	1.122
				0.15	0.8278	0.5289	0.5289	0.3035	0.3725	1.524	1.235	1.096
				0.2	0.8695	0.5333	0.5333	0.3356	0.3661	1.514	1.206	1.088
				0.4	1.031	0.5498	0.5498	0.4610	0.3395	1.476	1.097	1.060
				0.5	1.109	0.5574	0.5574	0.5220	0.3259	1.460	1.046	1.048
				0.6	1.185	0.5646	0.5646	0.5821	0.3122	1.444	0.9966	1.036
				0.8	1.334	0.5781	0.5781	0.6996	0.2844	1.416	0.9026	1.015
				1.0	1.477	0.5906	0.5906	0.8141	0.2562	1.390	0.8141	0.9963
				2.0	2.140	0.6423	0.6423	1.352	0.1128	1.294	0.4268	0.9251
1.8	0.5	0.6	0.7	0.1	0.6421	0.3155	0.4115	0.1856	0.3083	1.286	1.095	1.010
				0.15	0.7354	0.3414	0.4496	0.2285	0.3241	1.347	1.116	1.044
				0.4	1.080	0.3877	0.5524	0.4182	0.3273	1.451	1.047	1.048
				0.6	1.303	0.3616	0.6000	0.5578	0.2993	1.469	0.9388	0.9526
				1.1	1.788	1.425	0.6786	0.8857	0.1938	1.467	0.6373	2.484
		0.8	0.7	0.1	0.6795	0.3249	0.4436	0.1991	0.3212	1.335	1.108	1.010
2	0.5	0	0.5	0.15	0.6650	0.2975	0.3994	0.1598	0.3454	1.304	1.207	1.232 *
				0.4	1.024	0.3712	0.4955	0.3368	0.3508	1.377	1.122	1.308
				0.6	1.253	0.4063	0.5412	0.4711	0.3209	1.381	0.9984	1.318
			0.7	0.05	0.4766	0.2661	0.3103	0.1097	0.2573	1.[illegible]7	0.9992	0.9364
				0.15	0.6893	0.3432	0.3991	0.1906	0.3082	1.261	1.096	1.065
				0.4	1.043	0.4276	0.4956	0.3617	0.3196	1.352	1.040	1.131
				0.6	1.276	0.4677	0.5414	0.4884	0.2988	1.367	0.9445	1.140
				1.0	1.687	0.5235	0.6052	0.7282	0.2308	1.363	0.7282	1.134

*Convergence to 10^{-4}.

Table 4, cont.

β	ω	σ_1	Pr	t_w	f"(0)	θ'(0)	g'(0)	I_1	I_2	$I_1(1)$	$I_1(2)$	$I_1(3)$
2	0.5	0.25	0.7	0.05	0.4962	0.2672	0.3246	0.1172	0.2619	1.119	1.003	0.9319
				0.15	0.7137	0.3414	0.4167	0.2009	0.3119	1.287	1.096	1.053
				0.9452	1.669	0.0	0.6204	0.7180	0.2326	1.396	0.7348	0.3063
	0.7	0	0.5	0.15	0.7168	0.3333	0.4419	0.1673	0.3821	1.394	1.286	1.316
				0.4	1.049	0.3890	0.5157	0.3421	0.3646	1.418	1.150	1.347*
				0.6	1.276	0.4163	0.5523	0.4748	0.3268	1.403	1.010	1.339*
				1.0	1.687	0.4555	0.6052	0.7282	0.2308	1.363	0.7282	1.310
			0.7	0.05	0.5640	0.3270	0.3781	0.1264	0.3112	1.249	1.125	1.051
				0.15	0.7473	0.3851	0.4449	0.2028	0.3417	1.357	1.168	1.136
				0.4	1.071	0.4484	0.5179	0.3690	0.3326	1.396	1.067	1.164
				0.6	1.291	0.4793	0.5537	0.4930	0.3046	1.391	0.9561	1.158
				1.0	1.687	0.5235	0.6052	0.7282	0.2308	1.363	0.7282	1.134
		0.5	0.5	0.15	0.7679	0.2937	0.4734	0.2029	0.3621	1.421	1.217	1.193
				0.4	1.107	0.2994	0.5492	0.3899	0.3308	1.442	1.053	1.105
				0.6	1.345	0.2449	0.5865	0.5308	0.2838	1.427	0.8976	0.9170
	1	all	1	0	0.7386	0.5206	0.5206	0.1775	0.3837	1.544	1.288	1.111
				0.15	0.8972	0.5367	0.5367	0.2673	0.3626	1.506	1.187	1.082
				0.2	0.9483	0.5417	0.5417	0.2965	0.3553	1.495	1.156	1.074
				0.4	1.146	0.5601	0.5601	0.4101	0.3254	1.454	1.036	1.044
				0.5	1.241	0.5686	0.5686	0.4653	0.3101	1.436	0.9804	1.030
				0.6	1.333	0.5766	0.5766	0.5194	0.2945	1.420	0.9267	1.018
				0.8	1.513	0.5915	0.5915	0.6254	0.2629	1.390	0.8245	0.9957*
				1.0	1.687	0.6052	0.6052	0.7284	0.2309	1.363	0.7284	0.9760*
				2.0	2.488	0.6615	0.6615	0.1211	0.0676	1.263	0.3086	0.9023*

*Convergence to 10^{-4}.

Table 4, cont.

β	ω	σ_1	Pr	t_w	$f''(0)$	$\theta'(0)$	$g'(0)$	I_1	I_2	$I_1(1)$	$I_1(2)$	$I_1(3)$
2.4	1	all	1	0	0.7659	0.5244	0.5244	0.1611	0.3791	1.535	1.265	1.104
				0.5	1.335	0.5757	0.5757	0.4309	0.3005	1.422	0.9408	1.020 *
				1.0	1.838	0.6145	0.6145	0.6762	0.2151	1.347	0.6762	0.9637
2.8	1	all	1	0	0.7901	0.5275	0.5275	0.1480	0.3756	1.527	1.246	1.098
				0.5	1.421	0.5817	0.5817	0.4033	0.2928	1.411	0.9088	1.011 *
				1.0	1.978	0.6223	0.6223	0.6341	0.2023	1.333	0.6341	0.9537
3	0.5	0	0.7	0.15	0.7883	0.3503	0.4082	0.1616	0.3036	1.238	1.051	1.047
				0.4	1.228	0.4383	0.5093	0.3072	0.3074	1.323	0.9723	1.109*
				0.6	1.522	0.4805	0.5578	0.4142	0.2796	1.335	0.8603	1.115*
				1.0	2.044	0.5397	0.6258	0.6158	0.1967	1.328	0.6158	1.107
	0.7	0	0.7	0.05	0.6255	0.3324	0.3849	0.1057	0.3083	0.1231	1.091	1.037
				0.15	0.8493	0.3926	0.4545	0.1711	0.3360	1.334	1.121	1.118
				0.4	1.257	0.4593	0.5319	0.3125	0.3194	1.367	0.9973	1.141
				0.6	1.537	0.4923	0.5702	0.4176	0.2847	1.359	0.8708	1.133
				1.0	2.044	0.5397	0.6258	0.6158	0.1967	1.328	0.6158	1.107
3.4	1	all	1	0	0.8221	0.5314	0.5314	0.1325	0.3715	1.518	1.224	1.092
				0.5	1.541	0.5893	0.5893	0.3697	0.2835	1.396	0.8699	1.000
				1.0	2.170	0.6320	0.6320	0.5929	0.1919	1.317	0.5938	0.9416*
4	1	all	1	0	0.8502	0.5346	0.5346	0.1205	0.3684	1.511	1.207	1.086
				0.5	1.650	0.5955	0.5955	0.3434	0.2762	1.385	0.8393	0.9918
				1.0	2.347	0.6401	0.6401	0.5447	0.1751	1.305	0.5447	0.9320*
5	1	all	1	0	0.8907	0.5389	0.5389	0.1052	0.3647	1.502	1.184	1.079
				0.5	1.815	0.6042	0.6042	0.3096	0.2668	1.368	0.7994	0.9796
				1.0	2.616	0.6509	0.6509	0.4923	0.1586	1.288	0.4992	0.9196*

*Convergence to 10^{-4}.

Table 5

SIMILAR SOLUTIONS FOR A SUTHERLAND VISCOSITY—TEMPERATURE RELATION, Pr = 0.7, t_s = 1

This table is summarized on page 37.

β	s	f_w	σ_1	t_w	f"(0)	θ'(0)	g'(0)	I_1	I_2	$I_1(1)$	$I_1(2)$	$I_1(3)$	ω_r
0	0.01	0	0	0.15	0.3564	0.3100	0.3564	0.4105	0.3563	1.430	1.430	1.200	
					0.3554	0.3093	0.3554	0.4093	0.3554	1.425	1.425	1.195	0.5317
				0.4	0.4162	0.3643	0.4162	0.7909	0.4162	1.575	1.575	1.306	
					0.4161	0.3642	0.4161	0.7906	0.4161	1.574	1.574	1.306	0.5174
				0.6	0.4407	0.3868	0.4407	1.090	0.4406	1.628	1.628	1.344	
					0.4407	0.3868	0.4407	1.090	0.4406	1.628	1.628	1.344	0.5136
				1.0[a]	0.4696	0.4139	0.4696	1.686	0.4696	1.686	1.686	1.385	
					0.4696	0.4139	0.4696	1.686	0.4696	1.686	1.686	1.385	--
			0.5	0.15	0.3804	0.3019	0.3804	0.4681	0.3804	1.470	1.470	1.179	
					0.3797	0.3014	0.3797	0.4672	0.3798	1.467	1.467	1.176	0.5368
				0.6	0.4644	0.3247	0.4644	1.167	0.4643	1.664	1.664	1.240	
					0.4644	0.3247	0.4644	1.167	0.4643	1.664	1.664	1.240	0.5159
				0.9152	0.4871	0.0	0.4871	1.645	0.4871	1.710	1.710	0.7667	
					0.4871	0.0	0.4871	1.645	0.4871	1.710	1.710	0.7666	0.5121
			1	0.15	0.4326	0.3047	0.4326	0.5637	0.4327	1.540	1.540	1.149	
					0.4357	0.3050	0.4357	0.5558	0.4333	1.537	1.537	1.154	0.5486
				0.6	0.5126	0.2467	0.5126	1.285	0.5125	1.726	1.726	1.102	
					0.5191	0.2378	0.5191	1.239	0.5080	1.709	1.709	1.175	0.5205
				0.7	0.5198	0.1765	0.5198	1.438	0.5197	1.742	1.742	1.011	
					0.5266	0.1609	0.5266	1.384	0.5141	1.722	1.722	1.127	0.5184

[a] Solution independent of s.

Table 5, cont.

β	s	f_w	σ_1	t_w	f"(0)	$\theta'(0)$	$\phi'(0)$	I_1	I_2	$I_1(1)$	$I_1(2)$	$I_1(3)$	ω_r
0	0.01	-0.2	0	0.15	0.2221	0.2109	0.2221	0.5369	0.4221	1.824	1.824	1.514	
					0.2211	0.2101	0.2211	0.5355	0.4211	1.819	1.819	1.510	0.5317
				0.4	0.2794	0.2632	0.2794	0.9754	0.4794	1.922	1.922	1.577	
					0.2792	0.2631	0.2792	0.9751	0.4792	1.921	1.921	1.576	0.5174
				0.6	0.3028	0.2848	0.3028	1.318	0.5028	1.958	1.958	1.599	
					0.3028	0.2848	0.3028	1.318	0.5027	1.958	1.958	1.599	0.5136
				1.0[a]	0.3305	0.3108	0.3305	1.999	0.5305	1.999	1.999	1.623	
					0.3305	0.3108	0.3305	1.999	0.5305	1.999	1.999	1.623	--
		-0.6	0	0.15	0.0229	0.0376	0.0229	1.176	0.6228	4.137	4.137	3.483	
					0.0222	0.0368	0.0222	1.180	0.6221	4.154	4.154	3.439	0.5317
				0.4	0.0598	0.0802	0.0598	1.752	0.6598	3.437	3.437	2.809	
					0.0597	0.0800	0.0597	1.752	0.6597	3.438	3.438	2.810	0.5174
				0.6	0.0767	0.0982	0.0767	2.230	0.6768	3.299	3.299	2.672	
					0.0767	0.0982	0.0767	2.230	0.6767	3.299	3.299	2.672	0.5136
				1.0[a]	0.0975	0.1201	0.0975	3.181	0.6975	3.181	3.181	2.552	
					0.0975	0.1201	0.0975	3.181	0.6975	3.181	3.181	2.552	--
	0.03	0	1	0.4	0.4860	0.3052	0.4860	0.9686	0.4863	1.677	1.677	1.183	
					0.4939	0.3025	0.4939	0.9298	0.4854	1.667	1.667	1.211	0.5761
	0.05	0	1	0.15	0.4421	0.3134	0.4421	0.5759	0.4422	1.587	1.587	1.190	
					0.4452	0.3139	0.4452	0.5782	0.4451	1.587	1.587	1.187	0.7027
				0.4	0.4823	0.3045	0.4823	0.9640	0.4823	1.677	1.677	1.188	
					0.4890	0.3028	0.4890	0.9566	0.4865	1.677	1.677	1.200	0.6208
				0.6	0.5007	0.2492	0.5007	1.271	0.5007	1.716	1.716	1.112	
					0.5074	0.2418	0.5074	1.262	0.5052	1.716	1.716	1.134	0.5944
	0.1	0	0	0.15	0.4074	0.3559	0.4074	0.4577	0.4074	1.561	1.561	1.298	
					0.4011	0.3511	0.4011	0.4491	0.4011	1.529	1.529	1.270	0.7384
				0.4	0.4309	0.3778	0.4309	0.8095	0.4309	1.608	1.608	1.330	
					0.4299	0.3770	0.4299	0.8073	0.4299	1.603	0.603	1.326	0.6497

[a]Solution independent of s.

Table 5, cont.

β	s	f_w	σ_1	t_w	f"(0)	∂'(0)	g'(0)	I_1	I_2	$I_1(1)$	$I_1(2)$	$I_1(3)$	ω_r
0	0.1	0	0	0.6	0.4471	0.3928	0.4471	1.100	0.4471	1.641	1.641	1.354	
					0.4468	0.3926	0.4468	1.099	0.4468	1.640	1.640	1.353	0.6210
				1.0[a]	0.4696	0.4139	0.4696	1.686	0.4696	1.686	1.686	1.385	
					0.4696	0.4139	0.4696	1.686	0.4696	1.686	1.686	1.385	--
			0.5	0.15	0.4269	0.3393	0.4269	0.5168	0.4269	1.591	1.591	1.264	
					0.4224	0.3358	0.4224	0.5108	0.4223	1.570	1.570	1.246	0.7708
				0.4	0.4499	0.3422	0.4499	0.8761	0.4499	1.637	1.637	1.268	
					0.4495	0.3419	0.4495	0.8753	0.4495	1.635	1.635	1.267	0.6722
	0.3	0	0	0.15	0.4713	0.4140	0.4713	0.5150	0.4713	1.715	1.715	1.412	
					0.4615	0.4065	0.4615	0.5021	0.4615	1.668	1.668	1.371	0.9717
				0.4	0.4532	0.3983	0.4532	0.8373	0.4532	1.656	1.656	1.365	
					0.4513	0.3969	0.4513	0.8332	0.4513	1.648	1.648	1.358	0.8437
				0.6	0.4575	0.4024	0.4574	1.115	0.4573	1.663	1.663	1.367	
					0.4569	0.4020	0.4569	1.114	0.4569	1.660	1.660	1.367	0.7918
			0.5	0.15	0.4803	0.3823	0.4803	0.5722	0.4803	1.726	1.725	1.356	
					0.4738	0.3773	0.4738	0.5639	0.4738	1.697	1.697	1.333	1.019
				0.4	0.4631	0.3522	0.4631	0.8960	0.4630	1.671	1.670	1.291	
					0.4624	0.3517	0.4624	0.8947	0.462[illegible]	1.668	1.668	1.289	0.8837
				0.6	0.4676	0.3274	0.4676	1.177	0.4676	1.678	1.678	1.252	
					0.4677	0.3274	0.4677	1.177	0.4677	1.678	1.678	1.252	0.8266
				0.9151	0.4876	0.0	0.4876	1.645	0.4876	1.710	1.710	0.7664	
					0.4876	0.0	0.4876	1.645	0.4876	1.710	1.710	0.7664	0.5000
			1	0.15	0.4779	0.3416	0.4779	0.6203	0.4779	1.725	1.725	1.300	
					0.4792	0.3416	0.4792	0.6216	0.4792	1.723	1.723	1.296	1.101
				0.6	0.4723	0.2493	0.4735	1.238	0.4735	1.692	1.692	1.135	
					0.4788	0.2430	0.4788	1.240	0.4785	1.693	1.693	1.134	0.8838

[a] Solution independent of s.

Table 5, cont.

β	s	f_w	σ_1	t_w	f"(0)	θ'(0)	g'(0)	I_1	I_2	$I_1(1)$	$I_1(2)$	$I_1(3)$	ω_r
0	0.3	-0.2	0	0.15	0.3328	0.3116	0.3328	0.6259	0.5328	2.033	2.033	1.656	
					0.3227	0.3036	0.3227	0.6131	0.5227	1.984	1.984	1.613	0.9717
				0.4	0.3148	0.2959	0.3148	1.012	0.5149	1.979	1.979	1.613	
					0.3129	0.2944	0.3129	1.007	0.5129	1.971	1.971	1.605	0.8437
				0.6	0.3188	0.2998	0.3188	1.337	0.5188	1.983	1.983	1.614	
					0.3183	0.2994	0.3183	1.336	0.5183	1.980	1.980	1.612	0.7918
				1.0[a]	0.3305	0.3108	0.3305	1.999	0.5305	1.999	1.999	1.623	
					0.3305	0.3108	0.3305	1.999	0.5305	1.999	1.999	1.623	--
		-0.6	0	0.15	0.1005	0.1226	0.1005	1.018	0.7006	3.225	3.225	2.596	
					0.0914	0.1138	0.0914	1.015	0.6914	3.205	3.205	2.577	0.9717
				0.4	0.0858	0.1077	0.0858	1.675	0.6858	3.247	3.247	2.620	
					0.0841	0.1061	0.0841	1.675	0.6841	3.247	3.247	2.620	0.8437
				0.6	0.0886	0.1108	0.0886	2.187	0.6887	3.227	3.227	2.599	
					0.0882	0.1104	0.0882	2.187	0.6882	3.227	3.227	2.599	0.7918
				1.0[a]	0.0975	0.1201	0.0975	3.181	0.6975	3.181	3.181	2.552	
					0.0975	0.1201	0.0975	3.181	0.6975	3.181	3.181	2.552	--
0.4	0.05	0	1	0.15	0.5769	0.3342	0.4793	0.4341	0.4033	1.542	1.395	1.131	
					0.5808	0.3363	0.4854	0.4352	0.4063	1.545	1.394	1.128	0.7027
				0.4	0.7056	0.3268	0.5369	0.7259	0.4154	1.609	1.377	1.085	
					0.7130	0.3270	0.5496	0.7215	0.4203	1.610	1.380	1.093	0.6208
				1.1	0.9696	1.683	0.6169	1.495	0.3716	1.644	1.187	3.076	
					0.9706	1.819	0.6298	1.475	0.3793	1.639	1.202	2.732	0.5627

[a] Solution independent of s.

Table 5, cont.

β	s	f_w	σ_1	t_w	f''(0)	θ'(0)	g'(0)	I_1	I_2	$I_1(1)$	$I_1(2)$	$I_1(3)$	ω_r
0.5	0.01	0	0	0.15	0.4907	0.3422	0.3803	0.3909	0.2625	1.337	1.111	0.8468	
					0.4893	0.3466	0.3791	0.3907	0.2603	1.332	1.104	0.8389	0.5317
				0.4	0.6585	0.4208	0.4602	0.6170	0.2864	1.340	1.118	0.8351	
					0.6583	0.4207	0.4601	0.6167	0.2861	1.339	1.117	0.8342	0.5174
				0.6	0.7623	0.4691	0.5032	0.7846	0.2677	1.203	1.044	0.6484	
					0.7622	0.4691	0.5032	0.7844	0.2676	1.202	1.044	0.6482	0.5136
				1.0[a]	0.9278	0.4803	0.5420	1.147	0.3463	1.437	1.147	1.114	
					0.9278	0.4803	0.5420	1.147	0.3463	1.437	1.147	1.114	--
		-0.2	0	0.15	0.3647	0.2392	0.2554	0.3593	0.3822	1.648	1.512	1.356	
					0.3631	0.2383	0.2543	0.3582	0.3811	1.643	1.508	1.352	0.5317
				0.4	0.5333	0.3093	0.3321	0.6430	0.3963	1.661	1.447	1.340	
					0.5330	0.3092	0.3319	0.6427	0.3961	1.661	1.446	1.339	0.5174
				0.6	0.6367	0.3420	0.3688	0.8506	0.3935	1.644	1.383	1.332	
					0.6367	0.3420	0.3688	0.8505	0.3935	1.644	1.383	1.332	0.5136
				1.0[a]	0.8126	0.3815	0.4156	1.265	0.3793	1.684	1.265	1.378	
					0.8126	0.3815	0.4156	1.265	0.3793	1.684	1.265	1.378	--
		-0.6	0	0.15	0.1898	0.1104	0.0850	0.5001	0.2865	1.896	1.763	1.486	
					0.1884	0.1094	0.0842	0.4973	0.2867	1.890	1.763	1.489	0.5317
				0.4	0.3400	0.1553	0.1383	0.8302	0.5243	2.364	1.989	1.931	
					0.3398	0.1552	0.1381	0.8300	0.5241	2.364	1.989	1.931	0.5174
				0.6	0.4407	0.1854	0.1697	1.074	0.5031	2.275	1.813	1.847	
					0.4406	0.1854	0.1696	1.074	0.5031	2.275	1.813	1.847	0.5136
				1.0[a]	0.6138	0.2280	0.2147	1.532	0.4475	2.158	1.532	1.740	
					0.6138	0.2280	0.2147	1.532	0.4475	2.158	1.532	1.740	--
	0.02	0	0	0.2	0.5292	0.3517	0.4055	0.3575	0.3505	1.402	1.297	1.175	
					0.5273	0.3507	0.4042	0.3560	0.3493	1.397	1.292	1.170	0.5515

[a] Solution independent of s.

Table 5, cont.

β	s	f_w	σ_1	t_w	$f''(0)$	$\theta'(0)$	$g'(0)$	I_1	I_2	$I_1(1)$	$I_1(2)$	$I_1(3)$	ω_r
0.5	0.3	0	0	0.4	0.6968	0.4406	0.4964	0.6075	0.3695	1.491	1.297	1.150	
					0.6935	0.4321	0.4944	0.6037	0.3670	1.481	1.290	1.143	0.8437
				0.6	0.7740	0.4529	0.5108	0.7875	0.3615	1.469	1.242	1.136	
					0.7731	0.4525	0.5102	0.7863	0.3608	1.466	1.240	1.134	0.7918
				1.0[a]	0.9278	0.4803	0.5420	1.147	0.3463	1.437	1.147	1.114	
					0.9278	0.4803	0.5420	1.147	0.3463	1.437	1.147	1.114	--
		-0.2	0	0.15	0.4973	0.3495	0.3760	0.3989	0.4394	1.745	1.624	1.441	
					0.4793	0.3367	0.3612	0.3976	0.4509	1.771	1.630	1.450	0.9717
				0.4	0.5732	0.3424	0.369[illegible]	0.6606	0.4252	1.721	1.502	1.403	
					0.5698	0.3405	0.3668	0.6575	0.4235	1.715	1.496	1.398	0.8437
				0.6	0.6537	0.3543	0.3836	0.8653	0.4141	1.712	1.424	1.397	
					0.6527	0.3538	0.3930	0.8643	0.4136	1.710	1.423	1.396	0.7918
				1.0[a]	0.8126	0.3815	0.4156	1.265	0.3793	1.684	1.265	1.378	
					0.8126	0.3815	0.4156	1.265	0.3793	1.684	1.265	1.378	--
		-0.6	0	0.15	0.2762	0.1779	0.1587	0.5437	0.5998	2.587	2.306	2.073	
					0.2617	0.1685	0.1482	0.5362	0.5888	2.552	2.271	2.041	0.9717
				0.4	0.3682	0.1813	0.1644	0.8431	0.5466	2.396	2.008	1.941	
					0.3652	0.1795	0.1623	0.8410	0.5447	2.391	2.003	1.937	0.8437
				0.6	0.4530	0.1971	0.1815	1.080	0.5128	2.296	1.824	1.860	
					0.4522	0.1966	0.1810	1.080	0.5124	2.294	1.823	1.859	0.7918
				1.0[a]	0.6138	0.2274	0.2146	1.532	0.4477	2.162	1.532	1.750	
					0.6138	0.2274	0.2146	1.532	0.4477	2.162	1.532	1.750	--

[a]Solution independent of s.

Table 5, cont.

β	s	f_w	σ_1	t_w	f"(0)	θ'(0)	g'(0)	I_1	I_2	$I_1(1)$	$I_1(2)$	$I_1(3)$	ω_r
1	0.01	0	0.9	0.15	0.6889	0.3577	0.5201	0.2919	0.2376	0.4913	0.9650	0.7918	
					0.6884	0.3577	0.5199	0.2913	0.2342	0.4804	0.9574	0.7837	0.5450
				0.4	0.9225	0.3573	0.5768	0.5547	0.3678	0.567	1.179	1.041	
					0.9232	0.3578	0.5779	0.5551	0.3683	1.569	1.180	1.041	0.5249
				0.6	1.080	0.2915	0.6227	0.7333	0.3466	1.598	1.092	0.8959	
					1.077	0.2906	0.6196	0.7319	0.3455	1.593	1.090	0.8952	0.5192
				0.8	1.216	0.0135	0.6493	0.9023	0.3135	1.597	0.9885	0.4309	
					1.216	0.0121	0.6505	0.9026	0.3136	1.600	0.9885	0.4292	0.5158
		-0.2	1	0.4	1.825	0.8582	1.720	0.1881	0.1057	0.4019	0.3958	0.3462	
					1.975	0.9369	1.878	0.1740	0.0978	0.3703	0.3655	0.3192	0.5267
	0.05	0	0	0.4	0.8176	0.4184	0.4825	0.4682	0.3494	1.424	1.180	1.187	
					0.8162	0.4179	0.4817	0.4675	0.3488	1.422	1.179	1.185	0.5811
				0.6	0.9664	0.4498	0.5181	0.6296	0.3368	1.434	1.106	1.192	
					0.9660	0.4496	0.5179	0.6294	0.3366	1.433	1.106	1.192	0.5644
	0.1	0	0	0.15	0.6795	0.4550	0.4566	0.3003	-0.1013	1.247	0.3296	0.0345	
					0.6715	0.4555	0.4521	0.2981	-0.1471	1.183	0.2247	-0.0863	0.7384
				0.4	0.8784	0.5543	0.5113	0.5064	-0.2382	0.9784	-0.0073	-0.8562	
					0.8780	0.5580	0.5119	0.5064	-0.2592	0.9441	-0.0498	-0.9269	0.6497
				0.6	1.021	0.6526	0.5580	0.6602	-0.2510	0.5849	-0.0737	-1.835	
					1.021	0.6553	0.5589	0.6601	-0.2595	0.5598	-0.0905	-1.877	0.6210
	0.2	0	0	0.5	1.686	0.4311	0.6205	0.5346	0.0818	1.309	0.4055	0.9468	
					1.686	0.4312	0.6207	0.5347	0.0816	1.310	0.4051	0.9469	0.7805
		-0.5	0	0.05	0.4925	0.2520	0.2712	0.2870	0.5333	2.307	1.879	1.845	
					0.4496	0.2315	0.2458	0.2749	0.5099	2.216	1.791	1.764	1.0426

Table 5, cont.

β	s	f_w	σ_1	t_w	$f''(0)$	$\theta'(0)$	$g'(0)$	I_1	I_2	$I_1(1)$	$I_1(2)$	$I_1(3)$	ω_r
1	0.3	0	0	0.4	0.8554	0.4476	0.5140	0.4848	0.3705	1.489	1.226	1.235	
					0.8512	0.4458	0.5117	0.4824	0.3687	1.482	1.220	1.229	0.8437
				0.6	0.9925	0.4965	0.5375	0.6520	0.2582	1.370	0.9425	0.7262	
					0.9915	0.4964	0.5372	0.6511	0.2566	1.362	0.9388	0.7192	0.7918
			0.9	0.15	0.7397	0.3716	0.5242	0.3452	0.3945	1.583	1.339	1.169	
					0.7351	0.3695	0.5215	0.3430	0.3920	1.573	1.330	1.161	1.080
				0.6	1.031	0.2743	0.5597	0.7070	0.3244	1.486	1.061	0.8859	
					1.035	0.2750	0.5682	0.7088	0.3262	1.501	1.063	0.8842	0.8694
2	0.01	0	0	0.6	1.334	1.031	0.5433	0.5452	0.2251	1.272	0.7623	0.5427	
					1.334	1.030	0.5433	0.5451	0.2253	1.272	0.7629	0.5446	0.5136

Table 6

SOLUTION OF THE OUTER LIMIT EQUATIONS FOR $\beta \to \infty$

$I_1 \equiv 0$

ω	f_w	t_s	σ	$\left(\frac{u_e}{u_\infty}\right)^2$	Pr	t_w	$g'(0)$	$\theta'(0)$	I_2	$I_1(1)$	$I_1(2)$	$I_1(3)$
0.5	0	0.1	0.9	1	0.7	0.4	1.128	0.8388	-2.487	0.8330	-3.856	0.6067
						0.6	1.249	0.9283	-3.383	0.8262	-5.055	0.5952
			1	0	0.7	0.4	1.128	0.8388	-2.487	0.8330	-3.856	0.6067
						0.6	1.249	0.9283	-3.383	0.8262	-5.055	0.5952
				0.5	0.7	0.4	0.9489	0.8077	-3.008	0.9803	-4.641	0.6971
						0.6	1.050	0.8939	-4.056	0.9771	-6.043	0.6876
		0.3333	0.9	1	0.7	0.15	0.4837	0.4421	-0.0764	1.269	-0.1279	0.9456
						0.4	0.6181	0.5650	-0.5016	1.344	-0.9260	0.9786
						0.6	0.6840	0.6253	-0.8623	1.348	-1.529	0.9723
			1	0	0.7	0.15	0.6535	0.5016	-0.0039	0.9708	0.0087	0.7648
						0.4	0.8351	0.6410	-0.3297	1.011	-0.6133	0.7824
						0.6	0.9242	0.7094	-0.6099	1.007	-1.085	0.7734
				0.5	0.7	0.15	0.5495	0.4758	-0.0485	1.129	-0.0761	0.8555
						0.4	0.7022	0.6080	-0.4268	1.189	-0.7698	0.8821
						0.6	0.7772	0.6729	-0.7489	1.190	-1.329	0.8750
		1	0	--	0.7	0.15	0.4965	0.4154	0.3019	1.082	0.7833	0.9215
						0.4	0.6345	0.5309	0.2538	1.140	0.5792	0.9653
						0.6	0.7022	0.5875	0.1795	1.143	0.3866	0.9666
						1.0	0.7979	0.6676	0.0	2.128	0.0	1.954
			0.5	--	0.7	0.15	0.4175	0.3789	0.3310	1.262	0.8588	1.010
						0.4	0.5336	0.4842	0.2783	1.342	0.6350	1.058
						0.6	0.5905	0.5359	0.1968	1.351	0.4239	1.060
						1.0	0.6709	0.6089	0.0	2.342	0.0	2.046

Table 6, cont.

ω	f_w	t_s	σ	$\left(\frac{u_e}{u_\infty}\right)^2$	Pr	t_w	g'(0)	θ'(0)	I_2	$I_1(1)$	$I_1(2)$	$I_1(3)$
0.5	0	1	0.9	--	0.7	0.15	0.2792	0.2734	0.4587	1.853	1.190	1.400
						0.4	0.3568	0.3494	0.3856	1.986	0.8800	1.467
						0.6	0.3949	0.3867	0.2726	2.009	0.5874	1.468
						1.0	0.4487	0.4394	0.0	3.007	0.0	2.449
	-0.2	1	0	--	0.7	0.15	0.3766	0.3306	0.3439	1.253	0.9003	1.059
						0.4	0.5129	0.4451	0.2804	1.273	0.6418	1.070
						0.6	0.5801	0.5014	0.1962	1.261	0.4232	1.058
						1.0	0.6751	0.5811	0.0	2.228	0.0	2.030
			0.5	--	0.7	0.15	0.3319	0.3080	0.3727	1.431	0.9748	1.147
						0.4	0.4469	0.4126	0.3047	1.473	0.6972	1.162
						0.6	0.5035	0.4640	0.2134	1.468	0.4603	1.151
						1.0	0.5836	0.5367	0.0	2.441	0.0	2.122
			0.9	--	0.7	0.15	0.2403	0.2360	0.4995	2.018	1.303	1.533
						0.4	0.3176	0.3118	0.4116	2.115	0.9411	1.568
						0.6	0.3555	0.3489	0.2840	2.123	0.6232	1.558
						1.0	0.4092	0.0795	0.0	3.104	0.0	2.524
	-0.4	1	0	--	0.7	0.15	0.2730	0.2551	0.3937	1.460	1.041	1.225
						0.4	0.4037	0.3664	0.3106	1.426	0.7134	1.189
						0.6	0.4689	0.4216	0.2149	1.394	0.4644	1.161
						1.0	0.5619	0.5001	0.0	2.338	0.0	2.114
			0.5	--	0.7	0.15	0.2557	0.2441	0.4214	1.630	1.113	1.309
						0.4	0.3675	0.3463	0.3344	1.621	0.7676	1.279
						0.6	0.4230	0.3969	0.2318	1.597	0.5008	1.252
						1.0	0.5019	0.4687	0.0	2.548	0.0	2.205

Table 6, cont.

ω	f_w	t_s	σ	$\left(\frac{u_e}{u_\infty}\right)^2$	Pr	t_w	$g'(0)$	$\theta'(0)$	I_2	$I_1(1)$	$I_1(2)$	$I_1(3)$
0.5	-0.4	1	0.9	--	0.7	0.15	0.2040	0.2012	0.5451	2.202	1.[illegible]31	1.684
						0.4	0.2804	0.2760	0.4399	2.255	1.008	1.680
						0.6	0.3181	0.3130	0.3066	2.246	0.6619	1.655
						1.0	0.3714	0.3652	0.0	3.207	0.0	2.604
0.7	0	0.1	0.9	1	0.7	0.15	0.6877	0.5268	-1.516	0.9729	-2.505	0.7355
						0.4	1.131	0.8412	-2.449	0.8233	-3.780	0.6049
						0.6	1.235	0.9182	-3.274	0.7998	-4.865	0.5833
			1	0	0.7	0.15	0.9269	0.6899	-1.347	0.8417	-2.231	0.6287
						0.4	1.131	0.8412	-2.449	0.8233	-3.780	0.6049
						0.6	1.235	0.9182	-3.274	0.7998	-4.865	0.5833
				0.5	0.7	0.15	0.8395	0.7152	-1.572	0.9169	-2.576	0.6679
						0.4	1.021	0.8698	-2.771	0.9058	-4.256	0.6494
						0.6	1.114	0.9486	-3.668	0.8835	-5.435	0.6290
		0.3333	0.9	1	0.7	0.15	0.5855	0.5355	-0.0812	1.209	-0.1396	0.8938
						0.4	0.7091	0.6483	-0.4520	1.211	-0.8341	1.050
						0.6	0.7719	0.7055	-0.7596	1.189	-1.345	0.8608
			1	0	0.7	0.15	0.6966	0.5357	-0.0101	1.036	-0.003[illegible]	0.8116
						0.4	0.8468	0.6500	-0.3338	1.024	-0.62[illegible]	0.7929
						0.6	0.9231	0.7081	-0.6048	0.9998	-1.075	0.7707
				0.5	0.7	0.15	0.6308	0.5467	-0.0559	1.129	-0.0920	0.8497
						0.4	0.7649	0.6623	-0.4041	1.126	-0.7475	0.8358
						0.6	0.8329	0.7209	-0.6938	1.104	-1.230	0.8147
		1	0	--	0.7	0.15	0.5450	0.4598	0.3313	1.180	0.8471	0.9966
						0.4	0.6582	0.5530	0.2637	1.185	0.6000	1.0000
						0.6	0.7153	0.5999	0.1831	1.168	0.3941	0.9853
						1.0	0.7979	0.6676	0.0	2.128	0.0	1.954

Table 6, cont.

ω	f_w	t_s	σ	$\left(\frac{u_e}{u_\infty}\right)^2$	Pr	t_w	$g'(0)$	$\theta'(0)$	I_2	$I_1(1)$	$I_1(2)$	$I_1(3)$
0.7	0	1	0.5	--	0.7	0.15	0.4931	0.4495	0.3389	1.289	0.8666	1.019
						0.4	0.5943	0.5406	0.2698	1.304	0.6138	0.8082
						0.6	0.6454	0.5865	0.1873	1.289	0.4032	1.008
						1.0	0.7191	0.6526	0.0	2.252	0.0	1.976
			0.9	--	0.7	0.15	0.3887	0.3810	0.3998	1.618	1.022	1.203
						0.4	0.4677	0.4582	0.3183	1.647	0.7242	1.207
						0.6	0.5075	0.4971	0.2210	1.634	0.4757	1.189
						1.0	0.5649	0.5531	0.0	2.594	0.0	2.151
	-0.2	1	0	--	0.7	0.15	0.4227	0.3729	0.3714	1.345	0.9575	1.126
						0.4	0.5355	0.4663	0.2898	1.317	0.6613	1.102
						0.6	0.5926	0.5133	0.1996	1.285	0.4304	1.076
						1.0	0.6751	0.5811	0.0	2.228	0.0	2.030
			0.5	--	0.7	0.15	0.3929	0.3663	0.3789	1.452	0.9768	1.149
						0.4	0.4941	0.4576	0.2958	1.435	0.6750	1.125
						0.6	0.5452	0.5036	0.2038	1.406	0.4354	1.098
						1.0	0.6190	0.5699	0.0	2.351	0.0	2.052
			0.9	--	0.7	0.15	0.3261	0.3208	0.4394	1.778	1.131	1.331
						0.4	0.4053	0.3983	0.3441	1.776	0.7847	1.308
						0.6	0.4451	0.4373	0.2374	1.749	0.5115	1.279
						1.0	0.5026	0.4934	0.0	2.692	0.0	2.227
	-0.4	1	0	--	0.7	0.15	0.3149	0.2941	0.4182	1.540	1.089	1.281
						0.4	0.4247	0.3863	0.3193	1.467	0.7311	1.219
						0.6	0.4807	0.4328	0.2182	1.416	0.4710	1.177
						1.0	0.5619	0.5001	0.0	2.338	0.0	2.114

Table 6, cont.

ω	f_w	t_s	σ	$\left(\frac{u_e}{u_\infty}\right)^2$	Pr	t_w	$g'(0)$	$\theta'(0)$	I_2	$I_1(1)$	$I_1(2)$	$I_1(3)$
0.7	-0.4	1	0.5	--	0.7	0.15	0.3031	0.2908	0.4255	1.644	1.107	1.303
						0.4	0.4025	0.3810	0.3252	1.582	0.7445	1.241
						0.6	0.4529	0.4265	0.2232	1.535	0.4798	1.200
						1.0	0.5258	0.4923	0.0	2.460	0.0	2.136
			0.9	--	0.7	0.15	0.2683	0.2653	0.4844	1.960	1.257	1.478
						0.4	0.3469	0.3421	0.3727	1.918	0.8523	1.420
						0.6	0.3865	0.3809	0.2553	1.874	0.5510	1.377
						1.0	0.4437	0.4368	0.0	2.797	0.0	2.309
1	0	0.1	1	1	1	0	0.9076	0.9076	-1.115	0.9629	-1.841	0.6826
						0.5	1.189	1.189	-2.871	0.7798	-4.278	0.5481
						1.0	1.339	1.339	-4.528	0.7030	-6.327	0.4929
		0.1538	1	1	1	0	0.8258	0.8258	-0.5459	1.051	-0.9319	0.7461
						0.5	1.075	0.075	-1.736	0.8575	-2.756	0.6034
						1.0	1.208	1.208	-2.885	0.7753	-4.265	0.5442
		0.3333	1	1	1	0	0.7063	0.7063	0.0364	1.210	0.1645	0.8615
						0.5	0.9028	0.9028	-0.5265	1.006	-0.9472	0.7098
						1.0	1.010	1.010	-1.104	0.9161	-1.832	0.6445
		0.625	1	1	1	0	0.6326	0.6326	0.2647	1.331	0.7220	0.9504
						0.5	0.7915	0.7915	-0.0202	1.130	-0.0382	0.7994
						1.0	0.8793	0.8793	-0.3317	1.037	-0.6223	0.7317
		1	1	--	1	0	0.5898	0.5898	0.3567	1.412	1.010	1.010
						0.5	0.7230	0.7230	0.1965	1.220	0.4327	0.8654
						1.0	0.7979	0.7979	0.0	1.128	0.0	0.7979

Table 7

SOLUTION OF THE INNER LIMIT
EQUATIONS FOR $3 \to \infty$

ω	$a^{(*)}$	$s_0'(0)$
0.5	0	1.1547
	0.2025	1.1672
	0.4050	1.1816
	0.4920	1.1885
	1.0000	1.2533
0.75	0	1.1547
	0.2025	1.1609
	0.4050	1.1679
	0.4920	1.1712
	1.0000	1.1991
.0	all	1.1547

*

$$a = \frac{U_\infty^2}{2H_e} \; \frac{u_e}{u_\infty}^2 \cdot \frac{\cos^2 \Lambda}{t}$$

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10. ABSTRACT

Tables of 1400 values that may be used to determine skin friction, heat transfer, and appropriate boundary-layer thickness parameters for reentry vehicles. Assuming a perfect homogeneous gas atmosphere, the values were computed to an accuracy of four significant figures in terms of eight parameters: positive pressure gradient, Mach number, temperature-viscosity law, leading-edge sweep, wall temperature, Prandtl number, local streamwise velocity, and mass transfer at the surface (positive only). The accuracy requirements were relaxed for very high accelerations. More than five-sixths of the values are presented for the first time; the rest were recomputed to the accuracy of the present program. A key to the tables displays graphically the solutions available. Numerical procedures are given in detail. For application to cases in which not all of the eight parameters are similar, the direct effects of the nonsimilar terms may be calculated by asymptotically expanding the full boundary-layer equations in terms of the departures from similarity.

11. KEY WORDS

Aerodynamics
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